

The integrals are

$$Aup = \int_{x_0}^{x_0+h} AI(t) dt \quad (1.1)$$

$$Adn = \int_{x_0}^{x_0-h} AI(t) dt = - \int_{x_0-h}^{x_0} AI(t) dt \quad (1.2)$$

The value of AI at the midpoint is

$$AI(x_0 + h/2) = AI(x_0) \pm (h/2) \frac{dAI}{dx} \bigg|_{x_0} \quad (1.3)$$

Where the plus is for Aup and the minus is for And.

$$\begin{aligned} Aup &= hAI(x_0 + h/2) + O(h^3) \\ &= h \left(AI(x_0) + (h/2) \times \frac{dAI}{dx} \bigg|_{x_0} \right) + O(h^3) \end{aligned} \quad (1.4)$$

$$\begin{aligned} Adn &= hAI(x_0 - h/2) + O(h^3) \\ &= -h \left(AI(x_0 + h) - (h/2) \times \frac{dAI}{dx} \bigg|_{x_0+h} \right) + O(h^3) \end{aligned} \quad (1.5)$$

The h values in (1.4) and (1.5) are both positive. Define x_0+h to be x_1 , and $h_1=-h$ so that (1.5) becomes

$$Adn = h_1 \left(AI(x_1) + (h_1/2) \times \frac{dAI}{dx} \bigg|_{x_1} \right) + O(h^3) \quad (1.6)$$

Define

$$h_{BE} = \frac{Endc - Begc}{N} \quad (1.7)$$

$$\int_{Begc}^{Endc} AI(t) dt = \sum_{i=0}^{N-1} \int_{Begc+ih}^{Begc+(i+1)h} AI(t) dt = h_{BE} \sum_{i=0}^{N-1} \left(AI(Begc + ih_{BE}) + (h_{BE}/2) \times \frac{dAI}{dx} \bigg|_{Begc+ih_{BE}} \right) + NO(h^3) \quad (1.8)$$