Theory of Quantum Hall Nematics

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Transport measurements on two-dimensional electron systems in moderate magnetic fields suggest the existence of a spontaneously orientationally ordered, compressible liquid state. We develop and analyze a microscopic theory of such a "quantum Hall nematic" (QHN) phase, predict the existence of a novel, highly anisotropic q^3 density-director mode, find that the $T = 0$ long-range orientational order is unstable to weak disorder, and compute the tunneling into such a strongly correlated state. This microscopic approach is supported and complemented by a hydrodynamic model of the QHN, which, in the dissipationless limit, reproduces the modes of the microscopic model.

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Recent transport measurements on high mobility two-dimensional (2D) electron systems [1] have exhibited a striking anisotropy for Landau level (LL) filling factors close to $\nu = N + 1/2$, with $N \geq 4$. A natural explanation of this anisotropy is the development of a local 1D charge density wave (CDW) "stripe" state, which exhibits *spontaneous* orientational order, that can be further pinned by weak crystal fields or in-plane magnetic fields [2]. Hartree-Fock studies had predicted the existence of inhomogeneous states [3–5], with corroboration coming from exact diagonalization [6] and DMRG [7] studies of small systems. Such inhomogeneous states of matter are ubiquitous in systems where there is a competition between a repulsive, long-range (Coulomb) interaction and an attractive, short-range (exchange) interaction.

While structurally similar to conventional CDWs in metals, in semiconductors the CDW is expected to be only weakly pinned to the underlying crystal because of the large disparity between the electronic and ionic densities. Therefore, in semiconductors a good starting point for the CDW state is the "quantum Hall smectic" (QHS) [8–12], in which translational and orientational orders are broken *spontaneously,* with crystalline fields acting as small perturbations.

Although the QHS is an interesting state, an orientationally ordered quantum Hall nematic (QHN) [8,13] liquid is sufficient to account for the observed transport anisotropies. Additional motivation for studying the QHN is the instability of a 2D smectic to arbitrarily weak thermal fluctuations [14], which restore translational order and asymptotically reduce it to a nematic. Also, at $T = 0$ sufficiently strong quantum fluctuations can unbind dislocations, driving quantum melting of the QHS into the QHN [15] and providing a continuous transition from the QHS to the isotropic, compressible ($\nu = N + 1/2$) electronic liquid [16].

A conceptually useful picture of the QHN is a QHS in the presence of a plasma of *free* dislocations of density ξ_d^{-2}. In conventional liquid crystals, where dislocations obey simple diffusive dynamics, this picture allows for a reliable derivation of the nematic hydrodynamics from that of the smectic [14,17]. However, the $T = 0$ quantum dynamics of dislocations in the QHS is complex and is yet to be understood, as it depends upon the dislocation's charge, statistics, and the nature of the QHS state [15]; these subtleties are not addressed in treatments which use formal duality arguments [18] or a Lorentz-invariant extension of the static model [13].

In this Letter we present a theory of the compressible quantum Hall nematic, which is guided by the chiral edge dynamics [9,11,12] of the local smectic layers. The result is a novel quantum rotor model with a "soft" kinetic energy, stemming from the compressibility of the QHN and the underlying noninertial LL dynamics. Further support for the validity of our model comes from a complementary analysis of the finite T hydrodynamics of a charged nematic in a magnetic field, which in the collisionless limit reproduces the microscopic dynamics.

Local orientational order of the QHN is described by a unit director field $\mathbf{n}(\mathbf{r})$ that we take to be normal to the layers of smectic crystallites (that are positionally decorrelated by unbound dislocations). On scales longer than ξ_d the QHN Hamiltonian for the orientational degrees of freedom has the standard form

$$H_N = \frac{1}{2}\int d^2r\,\{\chi^{-1}(\delta\hat{\rho})^2 + K_1(\nabla\cdot\hat{\mathbf{n}})^2 + K_3[\hat{\mathbf{n}}\times(\nabla\times\hat{\mathbf{n}})]^2 - (\mathbf{h}\cdot\hat{\mathbf{n}})^2\}, \tag{1}$$

where K_1 and K_3 are the Frank elastic constants [10], $\mathbf{h}$ is a symmetry-breaking field (e.g., the crystalline anisotropy or an in-plane magnetic field), χ is the compressibility of this metallic state (including Coulomb interactions replaces χ^{-1} by $\chi^{-1} + 2\pi e^2/q$), $\delta\hat{\rho}(\mathbf{r})$ is the local electronic density *operator,* and in a single LL we have justifiably ignored the kinetic energy.

The quantum dynamics of the problem is encoded in the commutation relation for the operators $\delta\hat{\rho}$ and $\hat{\mathbf{n}}$. This can be obtained by quantizing the classical dynamics of the edges of the local smectic layers (taken parallel to the

x axis). A fluctuation in the electron density $\delta\rho$ leads to a force $-\partial_x(\chi^{-1}\delta\rho)$ parallel to the layers, which in the high field limit is balanced by the Lorentz force $-eB\dot{u}$; a *uniform* compression along the stripe leads to a layer displacement $u\hat{\mathbf{y}}$, consistent with the fact that the LL-projected particle coordinates X and Y do not commute (equivalently encoded in the dynamics of chiral edge bosons [9,11,12]). Consequently, a *nonuniform* compression along the stripe rotates the stripe through an angle $\theta \equiv \partial_x u$ determined by $eB\dot{\theta} = \partial_x^2(\chi^{-1}\delta\rho)$. Quantizing this dynamics on scales longer than ξ_d [19] leads to the commutation relation

$$[\delta\hat{\rho}(\mathbf{r}), \hat{\mathbf{n}}(\mathbf{r}')] = i\ell^2\hat{\mathbf{z}} \times \hat{\mathbf{n}}(\mathbf{r}')\partial_x^2\delta(\mathbf{r} - \mathbf{r}'), \quad (2)$$

where $\ell = \sqrt{1/eB}$ is the magnetic length and in our units $\hbar = k_B = 1$. The corresponding QHN action $S_N = \int dt\, d^2r\, \mathcal{L}_N$ is specified by a novel $O(2)$ "soft" quantum rotor model, with a Lagrangian density

$$\mathcal{L}_N = (eB)\mathbf{L} \cdot (\hat{\mathbf{n}} \times \partial_t\hat{\mathbf{n}}) - \tfrac{1}{2}\{\chi^{-1}(\partial_x^2\mathbf{L})^2 + K_1(\nabla \cdot \hat{\mathbf{n}})^2 + K_3[\hat{\mathbf{n}} \times (\nabla \times \hat{\mathbf{n}})]^2 - (\mathbf{h} \cdot \hat{\mathbf{n}})^2\}, \quad (3)$$

where $\mathbf{L} = \hat{L}_z\hat{\mathbf{z}}$ is the effective angular momentum operator conjugate to $\hat{\mathbf{n}}$, with commutation relation $[\hat{L}_z(\mathbf{r}), \hat{\mathbf{n}}(\mathbf{r}')] = i\ell^2\hat{\mathbf{z}} \times \hat{\mathbf{n}}(\mathbf{r}')\delta(\mathbf{r} - \mathbf{r}')$ and defined by $\delta\hat{\rho} = \partial_x^2\hat{L}_z$. In addition to the first "Berry's phase" term associated with the conserved z component of the angular momentum, a notable feature of the QHN is that the kinetic energy (the second term) vanishes at long wavelengths, a property which can be traced to the noninertial 2D electron dynamics in a strong magnetic field.

Having defined the model, we can now assess the effect of fluctuations on the QHN. Parametrizing the director as $\mathbf{n} = (\sin\theta, \cos\theta)$, we compute the (imaginary) time-ordered orientational correlation function $C(\mathbf{r}, \tau) = \langle \mathcal{T}_\tau \theta(\mathbf{r}, \tau)\theta(\mathbf{0}, 0)\rangle$, whose Fourier transform $\tilde{C}(\mathbf{q}, \omega_n)$ at Matsubara frequency $\omega_n = 2\pi nT$ can be measured in depolarized dynamic light scattering experiments. We find

$$\tilde{C}(\mathbf{q}, \omega_n) = T\ell^4\chi^{-1}\frac{q_x^4}{\omega_n^2 + \epsilon_{\mathbf{q}}^2}, \quad (4)$$

with an anisotropic dispersion $\epsilon_{\mathbf{q}}$ for orientational fluctuations about a y-directed nematic state given by

$$\epsilon_{\mathbf{q}} = \pm\ell^2\chi^{-1/2}q_x^2\sqrt{K_1q_x^2 + K_3q_y^2 + h^2}, \quad (5)$$

exhibiting a line of nodes along the nematic order. We note that in contrast to conventional nematics [14], in the QHN this orientational mode remains *gapless* ($\epsilon_{\mathbf{q}} \sim q_x^2$) even in the presence of an ordering field $\mathbf{h} = h\hat{\mathbf{y}}$. For $h = 0$ and in a single elastic constant approximation ($K_1 = K_3 = K$) the spectrum is $\epsilon_{\mathbf{q}} \propto q_x^2|q|$ ($\propto q_x^2|q|^{1/2}$ with Coulomb interactions).

Standard analysis shows that at $T = 0$ the equal-time connected correlation function $C_c(\mathbf{r}, 0) \equiv C(\mathbf{0}, 0) - C(\mathbf{r}, 0)$ saturates at $C_c(\infty, 0) \equiv \theta_{\rm rms}^2 \approx c_1\ell^2/(\xi_d^3\sqrt{K\chi}) \equiv T_Q/K$ [$c_1 = O(1)$ constant], and therefore predicts that a 2D QHN exhibits true long-range order at $T = 0$, with the nematic order parameter $\psi_2 = \langle e^{2i\theta}\rangle$ reduced by quantum fluctuations from its $T = 0$ classical value of 1 to $e^{-2\theta_{\rm rms}^2}$. At low T, such that $T \ll T_Q \approx 15$ mK, $C_c(\mathbf{r}, 0)$ exhibits a plateau for $\xi_d < r < \xi_T \equiv \xi_d(T_Q/T)^{1/3}$, but (for $h = 0$) asymptotically crosses over to classical logarithmic growth $\sim \ln(r/\xi_T)$ characteristic of quasi-long-range order of 2D classical nematics. For $h \neq 0$ the order is long-ranged even at finite T, but is reduced from the quantum-renormalized $T = 0$ value of $\psi_2(0, h)$ down to $\psi_2(T, h) = \psi_2(0, h)R(T, h)$, with $R(T, h)$ a complicated function of the strong-coupling orientational pinning length $\xi_{h0} = K^{1/2}/h$ [20]. In the quantum regime, $T \ll T_h \equiv T_Q(\xi_d/\xi_{h0})^3 < T_Q$ (equivalently, $\xi_d < \xi_{h0} \ll \xi_T$), we find $R(T, h) \approx e^{-(T/T_0)^{3/2}}$, with $T_0 = (K^2T_h/c_2^2)^{1/3}$ [$c_2 \approx \zeta(3/2)/(3\pi^{3/2}) \approx 0.16$]. Such nonanalytic (in T) thermal suppression of nematic order contrasts strongly with the classical spin-wave prediction of *linear* reduction with T [8]. In the opposite, semiclassical $T_h \ll T < T_Q$ regime (equivalently, $\xi_d < \xi_T \ll \xi_{h0}$), $R(T, h) = f(\xi_T/\xi_h)$, with $f(y) \approx y^{\eta_2/2}$, $\eta_2 = 2T/\pi K$, and $\xi_h = \xi_T(\xi_{h0}/\xi_T)^{4/(4-\eta_2)}$ the semiclassical pinning length, strongly thermally renormalized upward from its strong-coupling value ξ_{h0}. Even in this semiclassical regime, quantum effects are observable with T entering through *both* $\eta_2(T)$ and the quantum UV cutoff $\xi_T(T)$, only crossing over to the conventional classical result for $T > T_Q$, corresponding to $\xi_T(T) < \xi_d$.

Next, we turn to the effects of quenched disorder on the QHN, with the dominant effects at long length scales coming from the random anisotropy field $\delta\mathbf{h}(\mathbf{r})$ arising from, e.g., crystal defects and sample roughness, both randomly pinning the nematic director. Since the disorder is static, in a linear "Larkin" approximation (valid at scales smaller than disorder persistence length ξ_Δ) the quantum dynamics drops out, and we can adopt many recent results on the effects of disorder on classical liquid crystals [21]. For $h = 0$ we find that for $d < 4$ the nematic state is unstable to infinitesimally weak quenched disorder; in 2D the local orientational order persists out to only the Larkin length $\xi_L \approx (2\pi)^{3/2}K/\Delta^{1/2}$, where Δ is the variance of the random orientational field $\delta h_x(\mathbf{r})\delta h_y(\mathbf{r})$. To determine the nature of the state on longer scales, full nonlinearity of the random field must be taken into account [19].

Tunneling can provide important spectroscopic information about the low-energy excitations. In order to calculate the tunneling current we need to construct an electron creation operator $\hat{\Psi}^\dagger$, satisfying $[\delta\hat{\rho}(\mathbf{r}), \hat{\Psi}^\dagger(\mathbf{r}')] = \hat{\Psi}^\dagger(\mathbf{r}')\delta(\mathbf{r} - \mathbf{r}')$; the operator,

$$\hat{\Psi}^\dagger(\mathbf{r}, t) = \frac{1}{\sqrt{2\pi\ell^2}}\, e^{-i\int_{\mathbf{r}'}[D(\mathbf{r}-\mathbf{r}')\hat{\theta}(\mathbf{r}', t) + \mathrm{Arg}(\mathbf{r}-\mathbf{r}')\ell^2\partial_x^2\hat{L}_z(\mathbf{r}', t)]}, \quad (6)$$

accomplishes this, with $D(\mathbf{r} - \mathbf{r}')$ a solution of $\ell^2\partial_x^2 D(\mathbf{r} - \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')$ and a vortex attachment factor ensuring the fermionic statistics. The tunneling I-V is determined by the Laplace transform of the local imaginary-time Green's function $G(\tau) = \langle \mathcal{T}_\tau \hat{\Psi}(\mathbf{r},0) \times \hat{\Psi}^\dagger(\mathbf{r},\tau)\rangle \propto e^{-\Phi(\tau)}$, with $\Phi(\tau) = \frac{\chi^{-1}}{2}\int \frac{d^2q}{(2\pi)^2}(1 - e^{-\epsilon_\mathbf{q}|\tau|})/\epsilon_\mathbf{q}$. For short-range interactions, the leading behavior is $\Phi(\tau) \approx (|\tau|/\tau_0)^{1/2}$ ($\tau_0 \equiv \frac{1}{2}\pi^3 K^{1/2}\chi^{3/2}\hbar\ell^2\xi_d$), independent of the ξ_τ/ξ_h ratio ($\xi_\tau^3 = \xi_d^3 T_Q \tau/\hbar$). The differential tunneling conductance dI/dV into the QHN is therefore suppressed relative to both the isotropic $\nu = 1/2$ Halperin-Lee-Read (HLR) state [22] and the QHS [11,12]. Coulomb interactions modify the large h limit, giving $\Phi(\tau) \sim |\tau|^{3/5}$, but leave the small h regime unchanged.

Although the above model of the QHN is quite appealing and *mathematically* self-contained, on general physical grounds a theory of a *compressible* QHN, based on a collective nematic orientational degree of freedom $\theta(\mathbf{r},t)$ *alone* is necessarily incomplete. This shortcoming is clear from the defining properties of the QHN state, a *compressible* anisotropic *liquid,* in which free dislocations and their bound states of vacancies and interstitials (bubbles) constitute additional low-energy quasiparticle degrees of freedom. This contrasts strongly with the QHS, where a $T = 0$ description in terms of collective phonon (chiral edge boson) degrees of freedom alone is physically consistent [8–12], although not the only possibility [15].

Consequently, we must extend above the QHN model to include these additional quasiparticle degrees of freedom. Having been unable to do so from first principles, we conjecture that the quasiparticle sector of the full model is described by the HLR Lagrangian, $\mathcal{L}_{\rm HLR}[\hat{\psi},\mathbf{a}]$, with $\hat{\psi}(\mathbf{r})$ and $\mathbf{a}(\mathbf{r})$ the composite fermion (CF) and statistical gauge fields [16], coupled to the nematic orientation collective degree of freedom $\hat{\mathbf{n}}(\mathbf{r})$ via

$$\mathcal{L}_{\rm int} = g_m Q_{ij} D_i^* \hat{\psi}^\dagger D_j \hat{\psi} + (g_s|\nabla\cdot\hat{\mathbf{n}}|^2 + g_b|\nabla\times\hat{\mathbf{n}}|^2)\hat{\psi}^\dagger\hat{\psi}\,. \qquad (7)$$

This coupling is dictated by symmetry, with the first term the anisotropic CF mass proportional to the nematic order parameter $Q_{ij} = n_i n_j - (1/2)\delta_{ij}$ [with $\psi_2 = 2(Q_{xx} + iQ_{xy})$], the second and third terms the coupling of the CF density to splay and bend, and $\mathbf{D} = \nabla - i\mathbf{a}$.

The success of the HLR state at $\nu = 1/2$ [16] suggests that it might also be a good starting point at higher partially filled LLs, $\nu \approx N + 1/2$. Spontaneous development of orientational order (facilitated by screening of the Coulomb interaction by lower LLs) then leads to an orientationally ordered HLR state. Such a state naturally coincides with the above picture of the compressible QHN as a melted QHS. The relation $\partial_x\rho_{i-v} - \partial_y\theta = b_d(\mathbf{r})$ between the interstitial-vacancy density, $\rho_{i-v} = \rho_i - \rho_v$ and dislocation density $b_d(\mathbf{r})$ is reminiscent of the relation $\nabla\times\mathbf{a} = \rho_{cf}$ [16]. This intriguingly suggests the identification of the gauge field with vacancies and interstitials [i.e., $\mathbf{a}(\mathbf{r}) = (\theta, \rho_{i-v})$], and CF fermions with dislocations, lending further support to the above conjecture.

There are numerous experimental consequences of the quasiparticle model described by $\mathcal{L}_{qp} = \mathcal{L}_{\rm HLR} + \mathcal{L}_{\rm int}$, Eq. (7). The presence of quasiparticles leads to frequency shifts and Landau-like damping of the nematic director dynamics, which should be observable in dynamic light scattering. They will also contribute a second tunneling channel, which, to lowest order, adds in parallel to the tunneling results found above. Nematic order in turn induces a spontaneous distortion of the CF Fermi surface [23], responsible for highly anisotropic transport observed in experiments [1,2]. Equation (7) leads to surface acoustic wave (SAW) propagation, phase shifts, and attenuation that are anisotropic [16]. The Lagrangian $\mathcal{L}_{qp}$ predicts elliptic cyclotron orbits of CF, seeing an effective field $B_* = B[1 - 2(\nu - N)]$ (for filling fraction ν near $N + 1/2$) that should be observable in peak spacing anisotropy of magnetic focusing experiments [24], as well as in the period of the SAW anomaly [16]. We expect quasiparticles to lead to a violation of the semicircle law, otherwise satisfied by the resistivity tensor ρ_{ij} [9,25]. The director $\hat{\mathbf{n}}$ fluctuations will also contribute to the CF self-energies, modifying them relative to those of the isotropic HLR state [16]. We leave the analysis of these and other related problems to a future publication [19].

We now turn to hydrodynamics, which complements and provides further support for our microscopic model of the QHN. Focusing on the Goldstone director mode $\hat{\mathbf{n}}$ and conserved particle (ρ) and momentum ($\rho\mathbf{v}$) densities (ignoring for simplicity the energy density), standard methods lead to a set of hydrodynamic equations for an orientationally ordered 2D liquid of charged rods in the presence of a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$, which to linear order are

$$\begin{aligned}
\partial_t\rho &= -\rho_0\nabla\cdot\mathbf{v}\,,\\
\partial_t v_x &= -\frac{1}{2m\rho_0}(1-\lambda)\partial_y\left(\frac{\delta H_N}{\delta\theta}\right) - \partial_x\left(\frac{\delta H_N}{\delta\rho}\right)\\
&\quad + \omega_c v_y - \gamma_{xx}v_x - \gamma_{xy}v_y + \nu_{11}\partial_x^2 v_x\\
&\quad + \nu_{33}\partial_y^2 v_x + (\nu_{12}+\nu_{33})\partial_x\partial_y v_y\,,\\
\partial_t v_y &= \frac{1}{2m\rho_0}(1+\lambda)\partial_x\left(\frac{\delta H_N}{\delta\theta}\right) - \partial_y\left(\frac{\delta H_N}{\delta\rho}\right)\\
&\quad - \omega_c v_x - \gamma_{yx}v_x - \gamma_{yy}v_y + \nu_{22}\partial_y^2 v_y\\
&\quad + \nu_{33}\partial_x^2 v_y + (\nu_{12}+\nu_{33})\partial_x\partial_y v_x\,,\\
\partial_t\theta &= \frac{1+\lambda}{2}\partial_x v_y - \frac{1-\lambda}{2}\partial_y v_x - \gamma_\theta\frac{\delta H_N}{\delta\theta}\,,
\end{aligned} \qquad (8)$$

where H_N is given by Eq. (1), $\omega_c \equiv eB/m$, ν_{ij}'s are viscosities, and γ_θ and $\gamma_{ij} = \gamma_0\delta_{ij} + \gamma_Q Q_{ij}$ are the orientational and translational (momentum) relaxation rates. The *reversible* coupling λ (determined by the microscopic physics [26], and generically expected [17] to approach 1^- near the QHN-QHS transition), encodes the competition between the director's rotational convection and shear alignment. As in conventional nematics, for $|\lambda| \geq 1$ the director will shear align at an angle ϕ_λ to a prescribed uniform shear flow $\mathbf{v} = y\hat{\mathbf{x}}$, with $\tan\phi_\lambda = \sqrt{(\lambda+1)/(\lambda-1)}$; for $|\lambda| < 1$, such a steady state is impossible and $\hat{\mathbf{n}}$ will tumble anisotropically (via instantons).

Analysis of Eqs. (9) in the collisionless and Galilean-invariant limit ($\nu_{ij} = \gamma_\theta = \gamma_{ij} = 0$) leads to four modes, only two of which (corresponding to a mixture of density ρ and nematic orientation θ) are hydrodynamic and which propagate with a dispersion

$$\omega_{\mathbf{q}} = \frac{\pm \ell^2}{2\chi^{1/2}}[(1+\lambda)q_x^2 + (1-\lambda)q_y^2] \times \sqrt{K_1 q_x^2 + K_3 q_y^2 + h^2}. \quad (9)$$

Analysis of the eigenmodes reveals that this novel q^3 mode corresponds to clockwise/counterclockwise director oscillations beating against density fluctuations. For $\lambda = 1$ this mode is *identical* to the microscopic, $T = 0$ dispersion in Eq. (5). This, together with the observation that the long time limit of hydrodynamic equations can be obtained from a Poisson bracket between ρ and θ, generalized to an arbitrary λ [19], suggests that the microscopic commutation relation, Eq. (2), must be similarly generalized to

$$[\delta\hat{\rho}(\mathbf{r}), \hat{\mathbf{n}}(\mathbf{r}')] = \frac{i}{2}\ell^2 \hat{\mathbf{z}} \times \hat{\mathbf{n}}(\mathbf{r}') \times [(1+\lambda)\partial_x^2 + (1-\lambda)\partial_y^2]\delta(\mathbf{r}-\mathbf{r}'), \quad (10)$$

with a corresponding replacement in the Lagrangian (3) of $\partial_x^2 \mathbf{L} \to \frac{1}{2}[(1+\lambda)\partial_x^2 + (1-\lambda)\partial_y^2]\mathbf{L}$. A notable feature of the dispersion (9) in the alignable $|\lambda| > 1$ regime are lines of nodes along $q_y/q_x = \pm\tan\phi_\lambda$, a property that should reflect itself in a variety of experiments. For instance, our tunneling result for $\lambda = 1$ and short-range interactions, $\Phi(\tau) \sim |\tau|^{1/2}$, will become $\sim|\tau|^{1/3}$ for $h = 0$ and $\lambda \neq 1$, and for $h \neq 0$ will become $\sim \log\tau$ ($|\lambda| < 1$) and $\sim \log^2\tau$ ($|\lambda| > 1$).

In the presence of dissipation, at sufficiently long scales the two density-director modes become overdamped. For the Galilean-invariant case ($\gamma_{ij} = 0$) the relaxation rates are $\omega_- = -iD_- q^4$ and $\omega_+ = -iD_+(K_1 q_x^2 + K_3 q_y^2)$, with $D_\pm$ diffusion coefficients, that are functions of parameters in Eq. (8) [19]. However, because the QHN exhibits finite conductivity, at sufficiently long scales momentum relaxation becomes important and the density and director dynamics decouple, leading to diffusive relaxation rates $\omega_\rho = iD_\rho q^2$ and $\omega_\theta = iD_\theta(K_1 q_x^2 + K_3 q_y^2)$.

Although the velocity modes are nonhydrodynamic in the presence of a magnetic field, their relaxation determines the linear longitudinal transport. In the isotropic, $\psi_2 = 0$ (HLR) state, this leads to the Drude model with an isotropic resistivity. In the QHN, $\psi_2 \neq 0$ and we find a resistivity anisotropy $(\rho_{xx} - \rho_{yy})/(\rho_{xx} + \rho_{yy}) = \psi_2 \gamma_Q/\gamma_0$ proportional to the nematic order parameter [8], making the h and T dependence of the order parameter experimentally accessible.

In summary, by combining detailed microscopic and hydrodynamic analyses, we have proposed a model for a QHN, and argued that this novel orientationally ordered electronic liquid crystal state may be realized in nearly half-filled high LLs. We predict the existence of a novel director-density mode, with a highly anisotropic q^3 dispersion, which remains gapless even in the presence of an ordering field. The nematic order is long ranged at $T = 0$, but is destroyed by thermal fluctuations and quenched disorder. Clearly, many interesting theoretical and experimental questions remain and now can be addressed in detail using the model presented here [19].

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