Electronic "Liquid Crystals": Novel Phases of Electrons in Two Dimensions

Collaborators:

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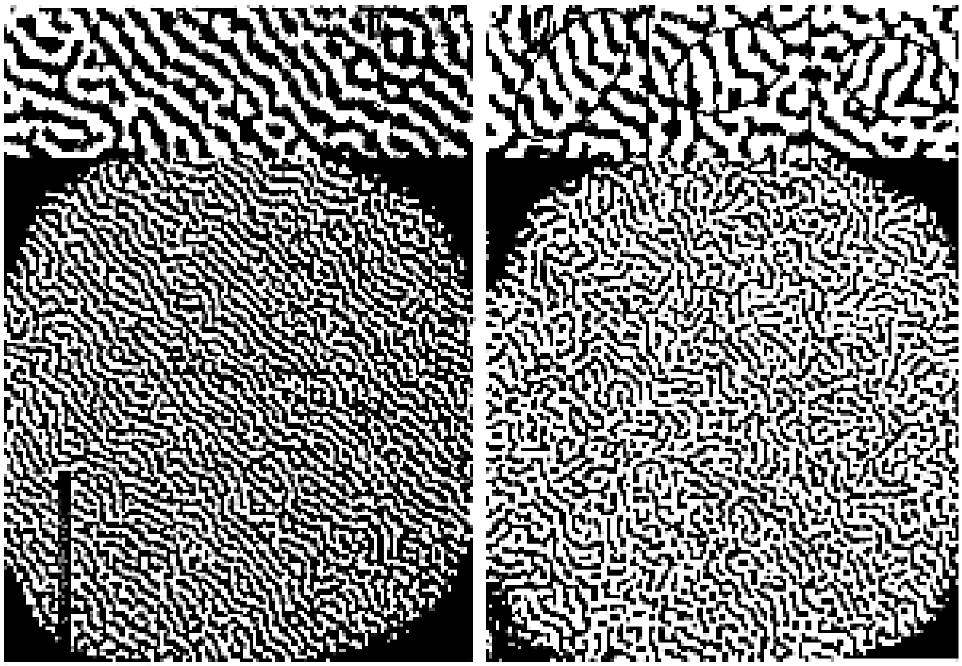
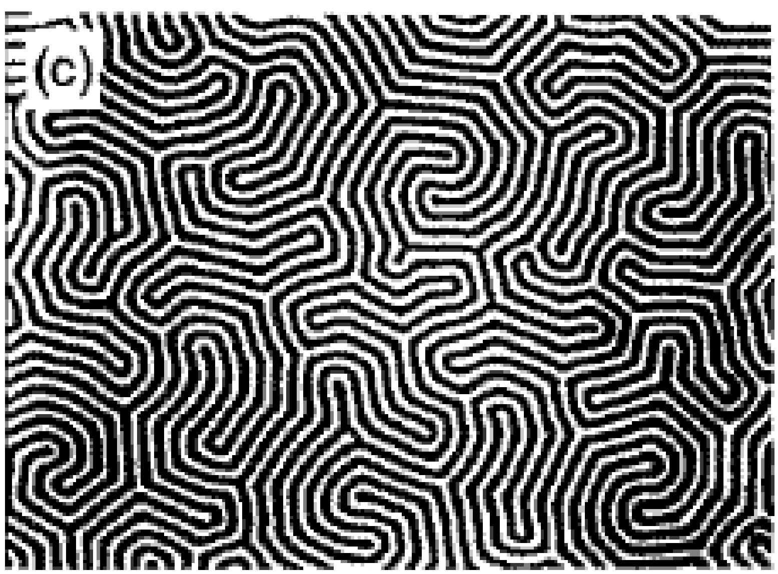
- C. Wexler & A. T. Dorsey, "Disclination unbinding transition in quantum Hall liquid crystals," Phys. Rev. B **64**, 115312 (2001).
- L. Radzihovsky & A. T. Dorsey, "Theory of Quantum Hall Nematics," cond-mat/0110083 (to appear in Phys. Rev. Lett.).

Modulated phases: examples

Langmuir monolayer (phospholipid and cholesterol):

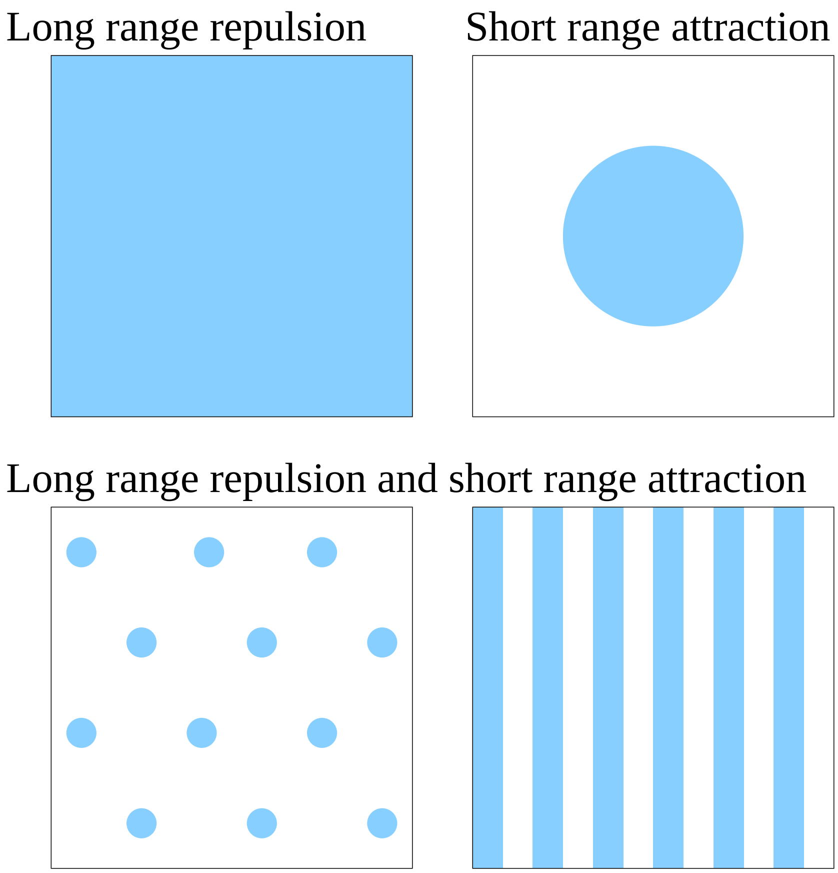


Thin ferromagnetic film (magnetic garnet):



Competing interactions

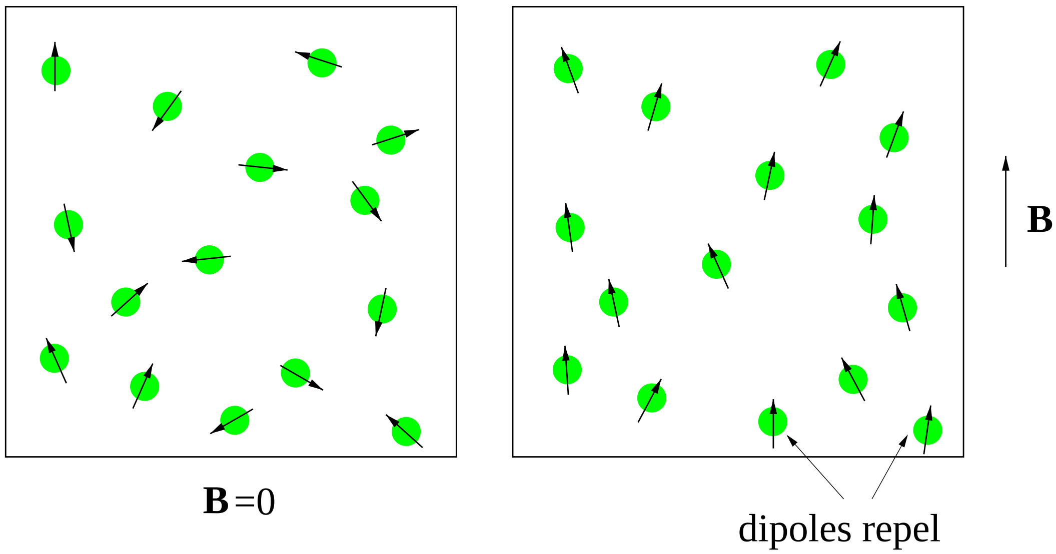
- Long range repulsive force: uniform phase.
- Short range attractive force (surface tension): compact structures.



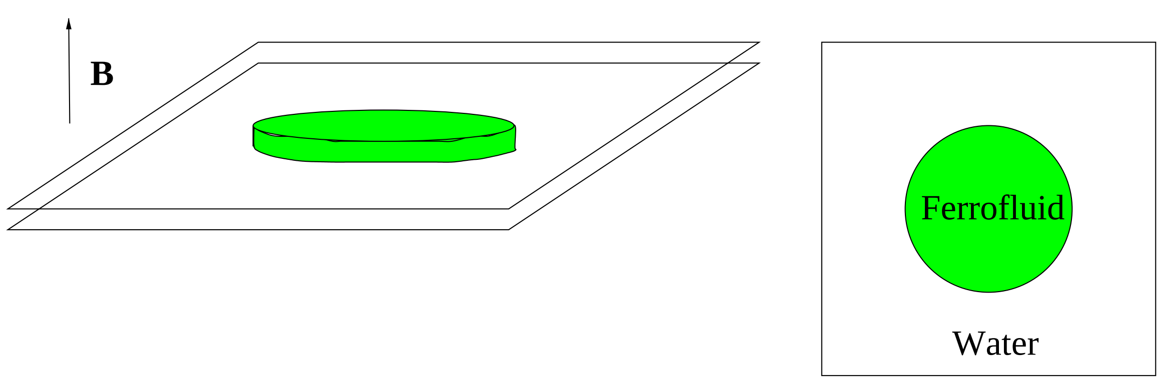
- Competition between forces ⇒ inhomogeneous phase. Observed in thin ferromagnetic films, ferrofluids, type-I superconductors, Langmuir monolayers, block copolymers, . . .

Demo: ferrofluid in a Hele-Shaw cell

Ferrofluid: colloid of 1 micron diameter spheres of magnetite. The fluid is paramagnetic and becomes magnetized in an applied field.



Hele-Shaw cell: ferrofluid between two glass plates.

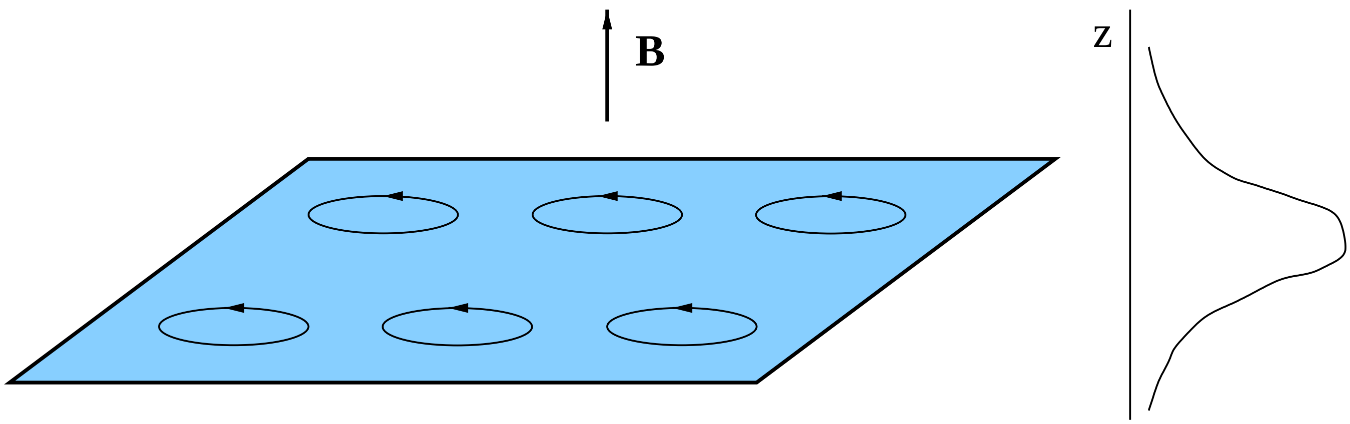


Surface tension (short range attraction) competes with repulsive dipole-dipole interaction.

Outline

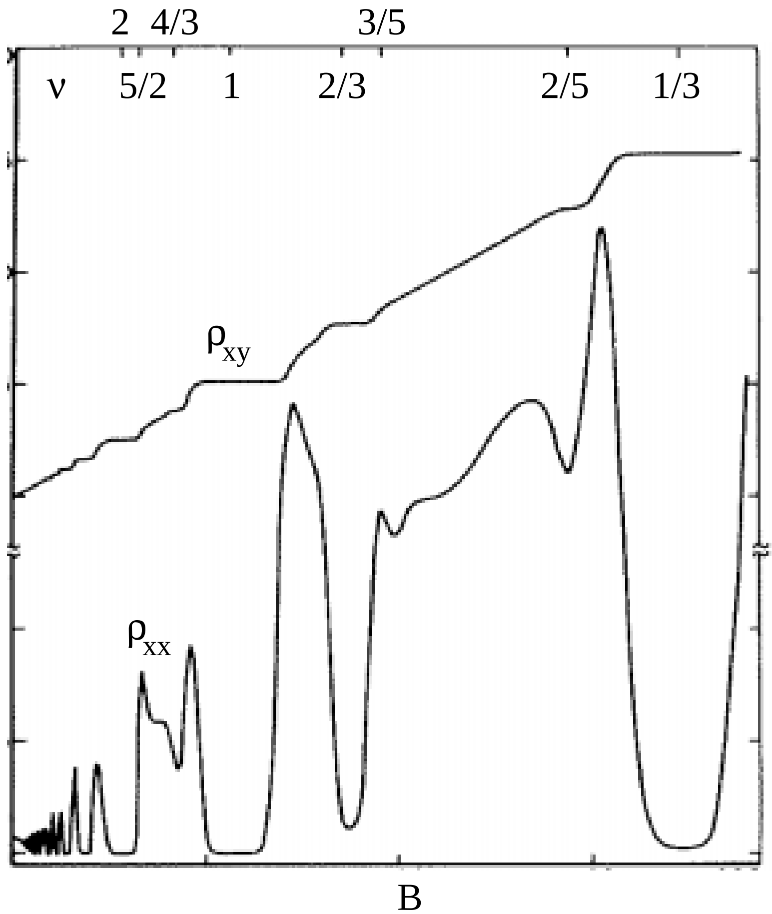
- Overview of the two dimensional electron gas and the quantum Hall effect.
- Theoretical and experimental evidence for a charge density wave?
- Liquid crystal physics in quantum Hall systems—smectics and nematics.
- Quantum theory of the nematic phase.

Two-dimensional electron gas (2DEG)



- Created in either Si MOSFETS or GaAs/AlGaAs heterostructures.
- Magnetic field quantizes electron motion into highly degenerate Landau levels (LL), $E_N = \hbar\omega_c(N+1/2)$. Cyclotron energy $\hbar\omega_c = \hbar eB/m^* = 19\,\mathrm{K/T}$ for GaAs ($m^* = 0.07m_e$).
- Magnetic length: $l_b = \sqrt{\hbar/eB} = 2.56 \times 10^{-6}\ \mathrm{cm/T}^{1/2}$.
- Coulomb energy: $E_C = e^2\sqrt{\pi n_e}/\epsilon \approx 100\ \mathrm{K}$ for an areal density of $n_e = 2.27\times 10^{11}\ \mathrm{cm}^{-2}$.
- Experiments at $k_B T \ll \hbar\omega_c,\ E_C,\ E_F$.

The quantum Hall effect



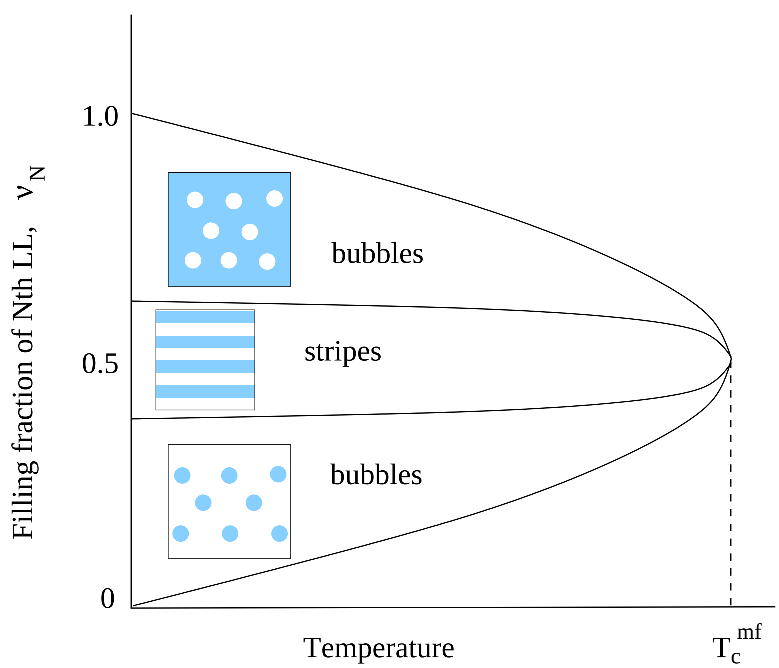
- Filling fraction (per spin):

$$\nu = \frac{\#\ \text{electrons}}{\#\ \text{states}} = \frac{N_e(h/e)}{BA} = \frac{hn_e}{eB}.$$

- $\sigma_{xy} = (e^2/h)\nu$, with $h/e^2 = 25,812\ \Omega$.
- Low disorder reveals intrinsic features of electron interactions. State of the art mobility $\sim 10^7\ \text{cm}^2/\text{V}\cdot\text{s}$.

Charge density waves in the 2DEG?

- CDWs were proposed as the ground state of a partially filled LL [Fukuyama, Platzman, & Anderson (1979)] but the Laughlin (liquid) state has lower energy. What happens in higher LLs (lower magnetic fields)?
- Using Hartree-Fock, Fogler, Koulakov, & Shklovskii (1996) predicted the existence of charge density wave (CDW) ground states for higher LLs. Shown to be exact for high LLs by Moessner and Chalker (1996).



Hartree-Fock treatment of the CDW

- Competition between direct & exchange ⇒ stripes or bubbles.

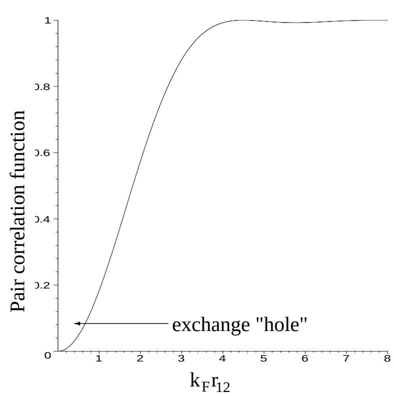
$$\epsilon_j\varphi_j(\mathbf{x}) = \hat{T}\varphi_j(\mathbf{x}) + \overbrace{\int_{\mathbf{y}} V(\mathbf{y}-\mathbf{x})\langle\hat{\psi}^\dagger(\mathbf{y})\hat{\psi}(\mathbf{y})\rangle\varphi_j(\mathbf{x})}^{\text{direct}-\text{“Hartree”}} \\ - \underbrace{\int_{\mathbf{y}} V(\mathbf{y}-\mathbf{x})\langle\hat{\psi}^\dagger(\mathbf{y})\hat{\psi}(\mathbf{x})\rangle\varphi_j(\mathbf{y})}_{\text{exchange}-\text{“Fock”}}.$$

- Direct: repulsive long range Coulomb interaction (projected onto a single LL):

$$V(\mathbf{q}) = \frac{e^2}{\epsilon l_b^2 q} e^{-l_b^2 q^2/2}\left[L_n^{n'-n}\left(l_b^2 q^2/2\right)\right]^2 \delta_{x_1-x_2+l_b^2 q_y}.$$

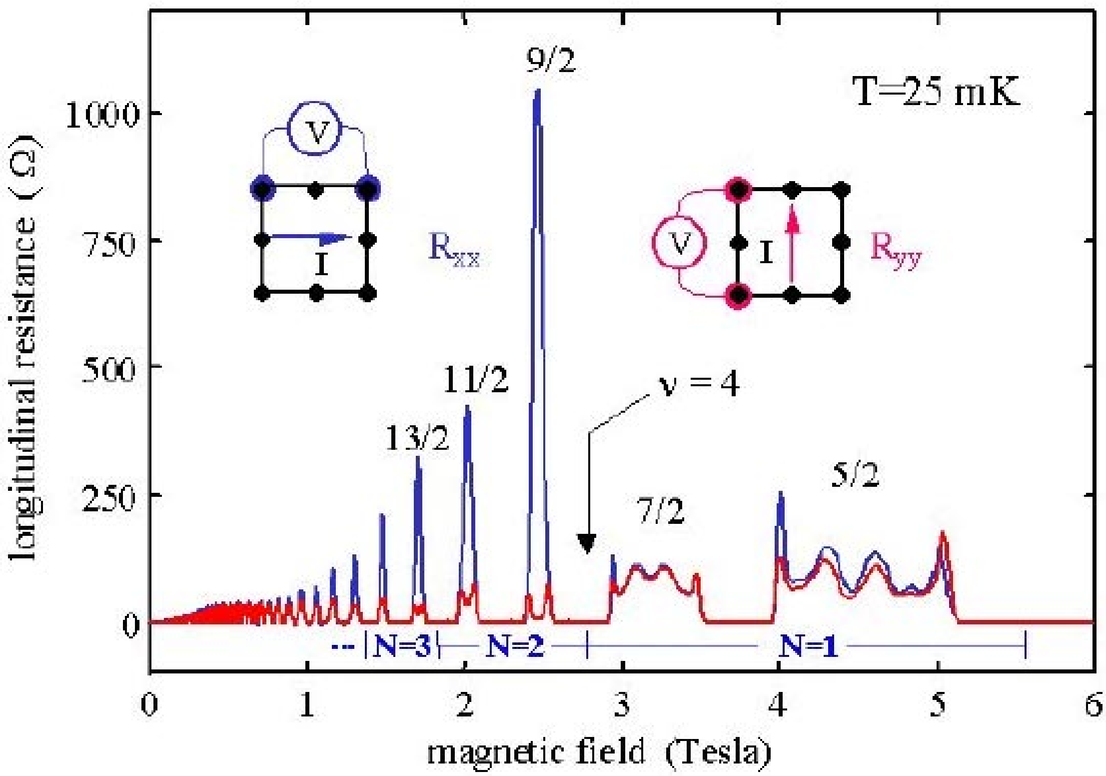
  Vanishes at the zeroes of the structure factors.

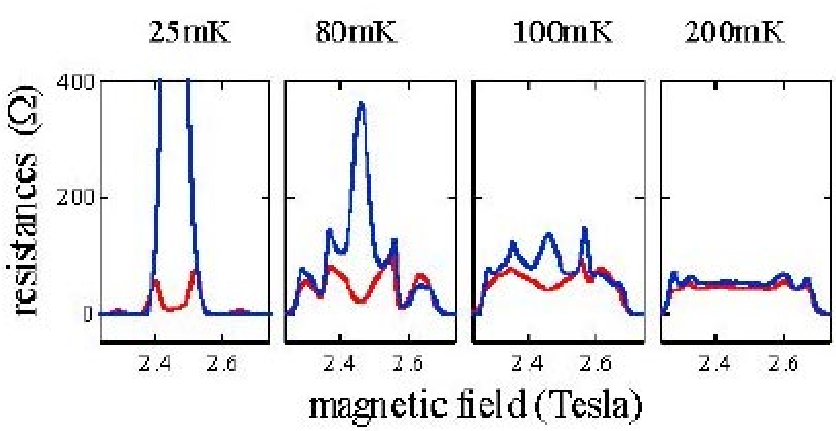
- Exchange: attractive short range interaction:



Experimental evidence

Anisotropic transport



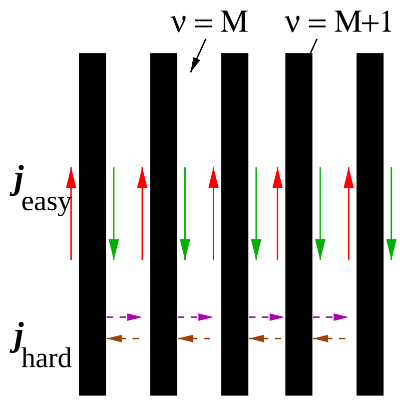


Some details

- Large anisotropy of ρ_{xx} in nearly half-filled LLs with $N \geq 2$ [Lilly *et al.* (1999), Du *et al.* (1999), Shayegan *et al.* (1999)]!
- Anisotropic-isotropic transition at 150 mK.
- Hard direction $[1\bar{1}0]$, easy direction $[110]$; "native" anisotropy energy $\sim$ 1 mK.
- Anisotropy can be oriented with an in-plane magnetic field.
- No QHE—compressible state.

A charge density wave?

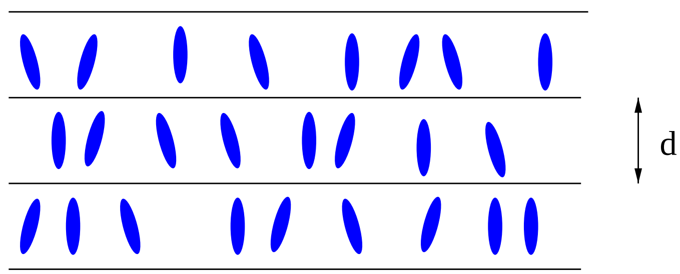
- Anisotropy in the transport is consistent with the formation of a unidirectional charge density wave (UCDW) state in the 2DEG.



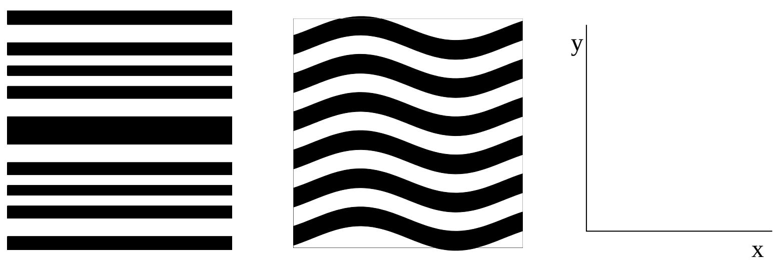
- Problems:
  - Transport in a static UCDW would be extremely anisotropic.
  - Formation energy of several K (vs. 150 mK transition temperature).
  - Data also consistent with an anisotropic liquid state.
- Fluctuations must be important [Fradkin and Kivelson (1999)].

The quantum Hall smectic

Classical smectic is a “layered liquid”:



- Fluctuations of the electronic stripes⇒ “quantum Hall smectic” [Fradkin & Kivelson (1999), MacDonald & Fisher (2000)].

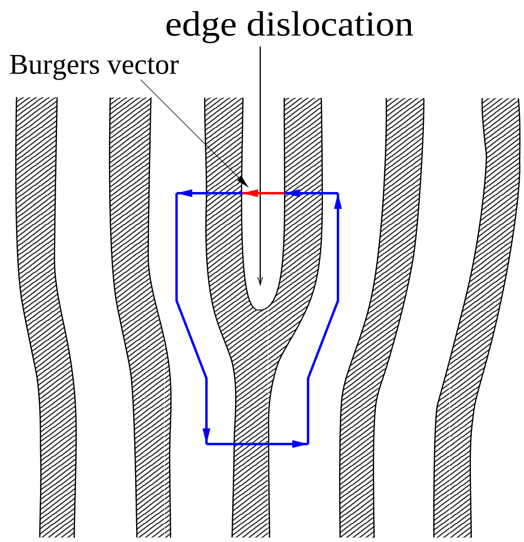


- C. Wexler & ATD (2001): find elastic properties from Hartree-Fock.

Problem: in $d = 2$ finite temperature phonons destroy the long-range positional order (Landau-Peierls), but preserve orientational order.

What about dislocations?

(Edge) dislocation = half a layer of atoms inserted into the crystal.



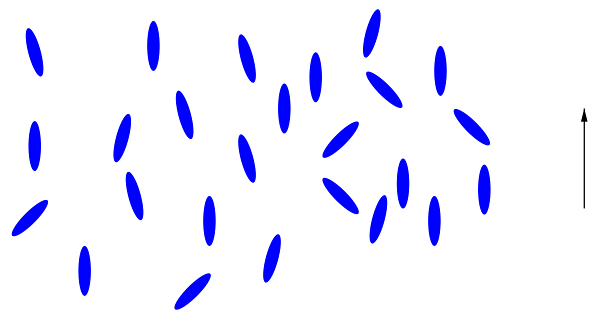
- Topological character:

$$\oint_\Gamma d\mathbf{u} = \mathbf{b}.$$

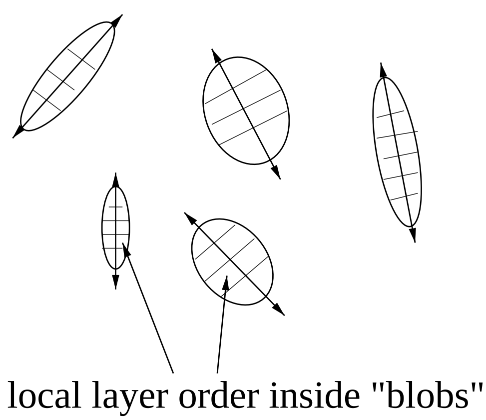
- Dislocation energy in a smectic is finite, so at temperature T there will be an areal density $n_D = \xi_D^{-2} \approx a^{-2} \exp(-E_D/k_B T)$.
- Dislocations further reduce the orientational order.

The quantum Hall nematic

- Classical nematic is an anisotropic liquid:



- Proliferation of dislocations "melt" the smectic, reducing the order to that of a nematic [Toner & Nelson (1982)].

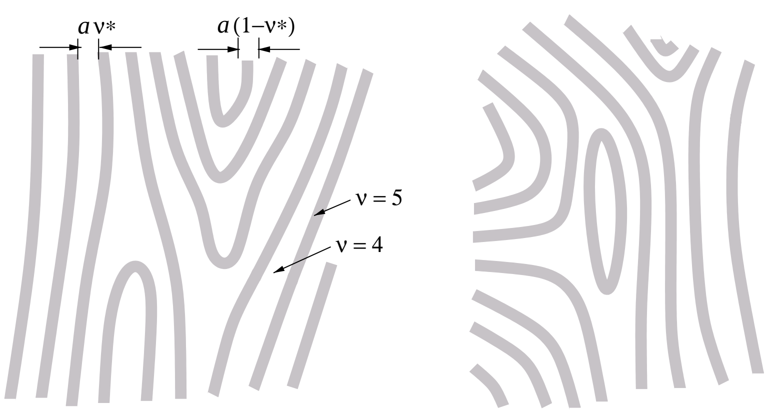


- The 2d nematic has algebraic orientational order (for $r \to \infty$):

$$\langle e^{i2\theta(\mathbf{r})} e^{-i2\theta(\mathbf{0})} \rangle \sim r^{-2k_B T/\pi K}.$$

Nematic → isotropic

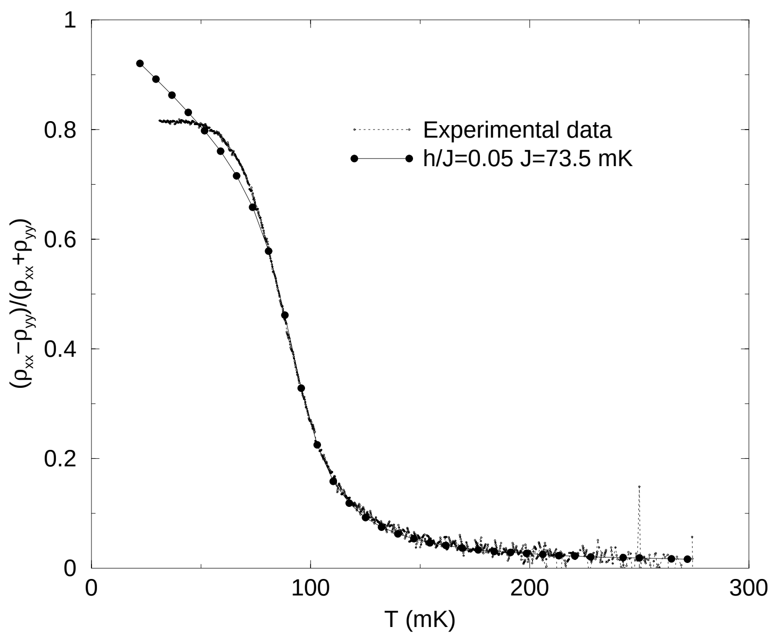
- Low temperature phase is better described as a nematic rather than as static stripes [Fradkin *et al.* (2000), Wexler & ATD (2001)].
- High temperature, isotropic phase is recovered through a disclination unbinding transition (Kosterlitz-Thouless).
- Wexler & ATD: calculate transition temperature starting from Hartree-Fock; obtain ~ 200 mK, experiments closer to ~ 100 mK. Obtain dependence on spin sub-band in agreement with experiments.



Quantum theory of the QHN

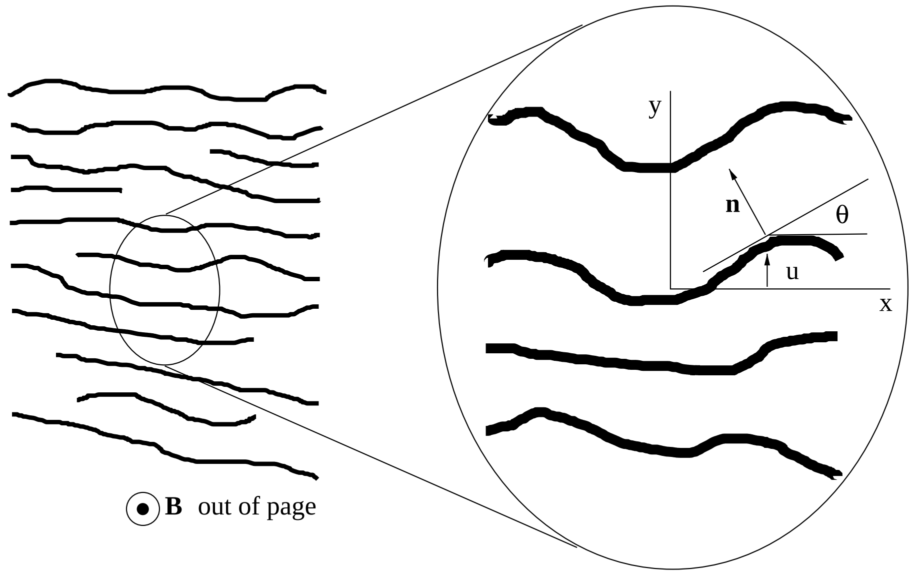
Why do we need a quantum theory?

- Classical theory overestimates the resistivity anisotropy below about 20 mK. Are quantum fluctuations the culprit?



- Quantum fluctuations can unbind dislocations, driving smectic into a $T = 0$ nematic phase [Fradkin & Kivelson (1999)].
- LR & ATD (cond-mat/0110083): use dynamics of local smectic layers as a guide. Make contact with hydrodynamics.

Dynamics for QHN



Nematic with dislocations

Dislocation-free region: local smectic order

- A fluctuation in the density $\delta\rho$ produces a force $-\partial_x(\chi^{-1}\delta\rho)$ parallel to the layers.
- In high magnetic fields this is balanced by the Lorentz force so that $-eB\dot{u} = -\partial_x(\chi^{-1}\delta\rho)$.
- Locally, a layer rotates through an angle $\theta \equiv \partial_x u \Rightarrow eB\dot{\theta} = \partial_x^2(\chi^{-1}\delta\rho)$.
- Assume that the entire nematogen rotates in the same fashion. For length scales $\gg \xi_D$,

$$[\delta\hat{\rho}(\mathbf{r}), \hat{\mathbf{n}}(\mathbf{r}')] = i\ell^2\hat{\mathbf{z}} \times \hat{\mathbf{n}}(\mathbf{r}')\partial_x^2\delta(\mathbf{r}-\mathbf{r}').$$

Theoretical bravado

- The angular momentum L_z and $\mathbf{n}$ are conjugate variables, with $\delta\rho = \partial_x^2 L_z$.
- The Lagrangian density is

$$\mathcal{L}_N = \underbrace{(eB)\mathbf{L}\cdot(\hat{\mathbf{n}}\times\partial_t\hat{\mathbf{n}})}_{\text{“Berry's phase” term}} - \frac{1}{2}\Bigg\{ \underbrace{\chi^{-1}(\partial_x^2\mathbf{L})^2}_{\text{“soft” kinetic energy}} + K_1(\boldsymbol{\nabla}\cdot\hat{\mathbf{n}})^2 + K_3(\boldsymbol{\nabla}\times\hat{\mathbf{n}})^2 - (\mathbf{h}\cdot\hat{\mathbf{n}})^2\Bigg\}.$$

  Quantum rotor model with “soft” kinetic energy. Can integrate out $\mathbf{L}$ to obtain nonlocal action in $\mathbf{n}$.
- Goldstone mode:

$$\epsilon_{\mathbf{q}} = \pm\ell^2\chi^{-1/2}q_x^2\sqrt{K_1q_x^2 + K_3q_y^2 + h^2}.$$

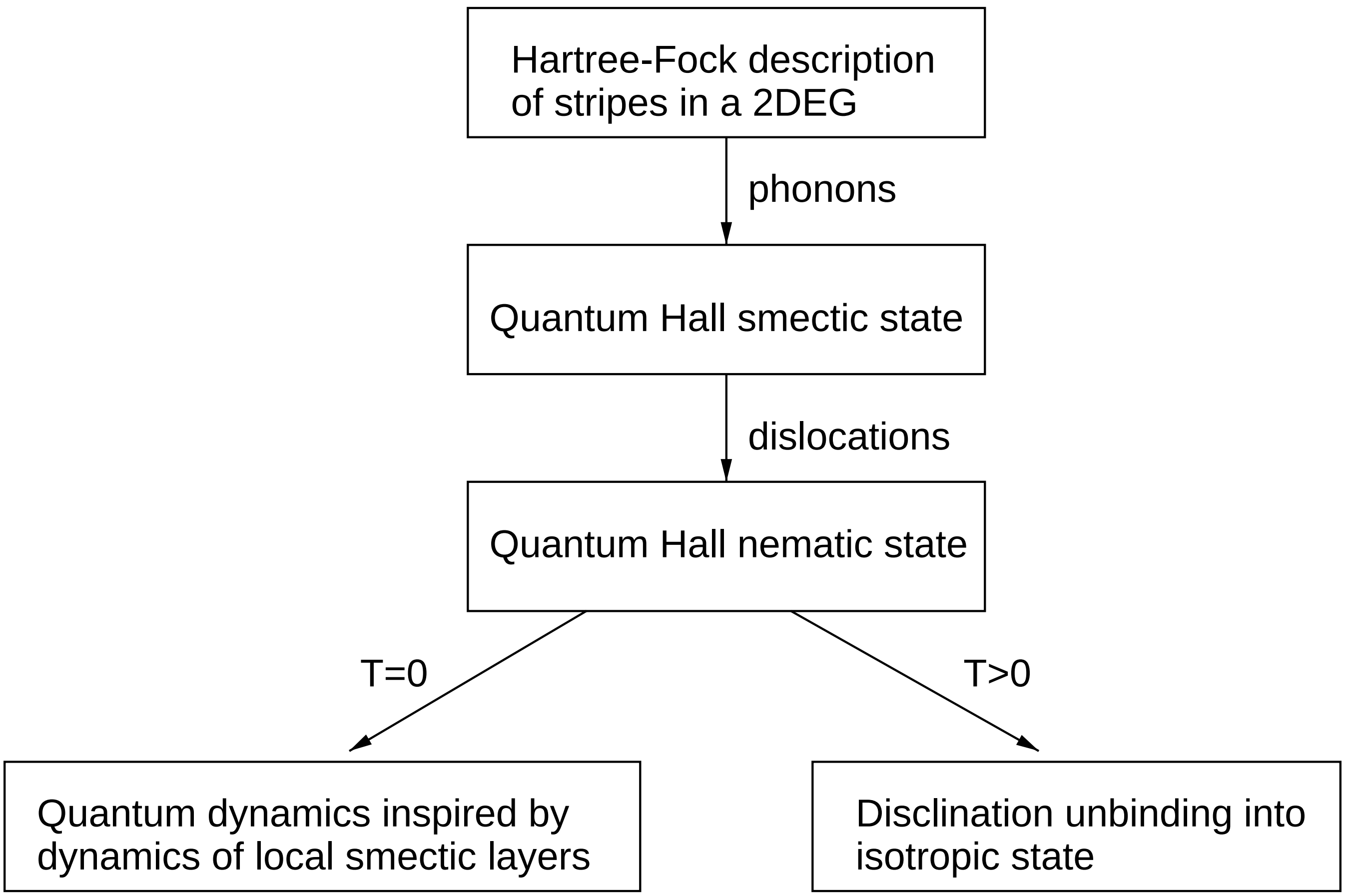
$\epsilon_{\mathbf{q}} \sim q^3$ for $h = 0$. Line of nodes along q_x.

Highlights and predictions

- Collective mode agrees with collisionless limit of $T > 0$ hydrodynamics (nematodynamics).
- QHN exhibits true LRO at $T = 0$. Nematic order parameter $\psi_2 \equiv \langle e^{2i\theta} \rangle \approx e^{-2\theta^2_{rms}} < 1$. Quantum fluctuations important for $T < T_Q \sim 20$ mK.
- QHN unstable to weak quenched disorder. Quantum nematic glass phase?
- Tunneling probes low energy excitations. $I(V) \sim \exp(-V_0/V) \Rightarrow$ "pseudogap". Tunneling suppressed relative to smectic.
- Damping of Goldstone mode due to coupling to quasiparticles [similar to V. Oganesyan *et al.* (2002)].
- Resistivity anisotropy proportional to nematic order parameter ψ_2, as conjectured by Fradkin *et al.* (2000).

Summary

Fascinating problem of orientationally ordered point particles!



Future directions: disorder, magnetic focussing experiments, relation to stripes in the cuprates, . . .