PHY 6346 Fall 2015

Homework #10, Due Wednesday, December 9

1. An electromagnetic device consists of two circular conducting plates of radius a, separated by a distance d (small compared to) a in the direction perpendicular to the plates. A slowly varying current $I = I_0 \cos \omega t$ is fed by a long wire into the center of one plate and removed by another long wire from the center of the other plate. Do calculations for this problem as real parts of complex quantities.

(a) What are the directions of the electric and magnetic fields between the plates?

(b) Find the electric and magnetic fields between the plates to leading order in ω.

(c) Find the time-average electric and magnetic energies contained within the device volume for the fields in (b).

(d) Find the capacitance and inductance at this order. Determine the resonant frequency (that for which $X = 0$).

(e) Find the time-average Poynting flux between the plates. What is the time-average energy entering (or leaving) the device volume? What happens if the material between the plates has a conductivity σ?

(f) Find the electric and magnetic fields inside the device including terms up to order ω^2. What values of ω are "slowly varying."

(g) Find the solution when ω is not necessarily small. (Start from Maxwell's equations. Bessel functions might appear.) Determine the lowest resonant frequency.

(h) To leading order for small ω, what is the magnetic field outside? (Ignore the plates; from now on, the rest is for the wire.) What is the electric field outside? How small must ω be now to be "small" or "slow"?

(i) What is the time-average Poynting flux just outside the wire?

(j) Find the exact vector potential outside the wire. It may or may not be useful to know

$$\int_{-\infty}^{\infty} d\xi \, \frac{e^{i\sqrt{\beta^2+\xi^2}}}{\sqrt{\beta^2+\xi^2}} = i\pi \, H_0^{(1)}(\beta).$$

Verify the fields near the wire.

Bonus: Find the fields far from the wire (what determines "far"?). Find the Poynting flux for large ρ. What is the angular distribution of power about the wire? What is the rate at which power is radiated per length of wire?