PHY 6346 Fall 2015

Homework #7, Due Monday, October 26

1. A spherical nucleus has constant charge density ρ_0 for $r < R_0$. The surface of the nucleus is distorted so that the radius becomes

$$R = R_0\,[1 + \alpha + \gamma\, P_2(\cos\theta)].$$

(a) Find the multipole moments of the distorted nucleus for small α, γ.

(b) For given γ, determine α so that the nuclear charge is unchanged.

(c) Write the potential outside the distorted sphere.

(d) Find the potential inside the distorted sphere, taking the exterior correction to arise from a surface $\sigma(\theta)$.

(f) What is the relative shift in energy due to the distortion?

(Bonus) Show that a $\beta\cos\theta$ term in the distortion can be removed by shifting the origin without changing other results.

2. A charge density $\rho(\boldsymbol{x})$ localized near the origin sits in an electrostatic potential $\Phi(\boldsymbol{x})$ that arises from distant sources.

(a) Show that the resulting electrostatic force on the charge distribution can be written as an expansion in multipole moments q, p_i, Q_{ij}, etc., of which the first few terms are

$$\begin{aligned}\boldsymbol{F} &= -(\nabla_i\Phi)_0\, q - \left(\nabla_i\nabla_j\Phi\right)_0\, p_j - \frac{1}{6}\,(\nabla_i\nabla_j\nabla_k\Phi)_0\, Q_{jk} + \cdots \\ &= q\,\boldsymbol{E}_0 + p_j\,(\nabla_j E_i)_0 + \frac{1}{6}\,Q_{jk}\,(\nabla_j\nabla_k E_i)_0 + \cdots\end{aligned}$$

where field and derivatives are evaluated at the origin. Show that the force on a dipole can be written variously as $\boldsymbol{F}_{\text{dipole}} = (\boldsymbol{p}\cdot\nabla)\boldsymbol{E} = \nabla(\boldsymbol{p}\cdot\boldsymbol{E})$.

(b) Show that the torque acting on the charge distribution can be written as

$$\boldsymbol{N} = \epsilon_{ijk}\,p_j\,E_k + \frac{1}{3}\,\epsilon_{ijk}\,Q_{jm}\,\nabla_m E_k + \cdots = \boldsymbol{p}\times\boldsymbol{E} + \cdots.$$

3. A point charge q is located in free space a distance d from the center of a dielectric sphere of radius a with permittivity $\epsilon = \kappa\epsilon_0$ (take $d > a$).

(a) Find the potential at all points in space as an expansion in Legendre polynomials. Comment on the behavior of your result as $\kappa \to 1$ and as $\kappa \to \infty$.

(b) Calculate the electric field near the center of the sphere. Compare with the result for a dielectric sphere in a uniform applied field (JDJ 4.55) in the limit $d \gg a$.

(c) Calculate the polarization charge density on the surface of the sphere.

(d) Calculate the force on the point charge q resulting from the polarization. How does this behave for $d \gg a$ (easy), and for $d \to a$ (harder). Compare the force as $d \to a$ to the force between a charge q and its image in a planar interface, as in JDJ Section 4.4

(e) When q is far from the sphere ($d \gg a$), use the induced dipole moment to compute the force that q exerts on the sphere. Compare with the result in (d) in this limit.