PHY 6346 Fall 2015

Homework #8, Due Monday, November 9

1. Find a vector potential that produces the magnetic field $\boldsymbol{B} = C(x\,\hat{\boldsymbol{x}} - y\,\hat{\boldsymbol{y}})$ and satisfies the condition $\nabla \cdot \boldsymbol{A} = 0$. Find another vector potential that produces the same magnetic field and also satisfies $\nabla \cdot \boldsymbol{A} = 0$.

2. Show how to write $E(-m)$ as an elliptic integral of positive argument. Show how to write $K(-m)$ as an elliptic integral of positive argument. What is the allowed range of positive arguments? What is the allowed range of negative arguments?

3. The surface current distribution on a disk of radius a has the form $\boldsymbol{K} = K_0\,\hat{\boldsymbol{\phi}}$, where K_0 is constant.

(a) Find the magnetic field on the z-axis.

(b) Find the magnetic field everywhere for $r > a$.

(c) Find the vector potential near the z-axis.

(d) Use the Bessel Function representation of the Green's function to find the vector potential and magnetic field near the surface of the disk. In particular, find the radial magnetic field at $z \to 0$ and compare with what you obtain from Ampére's law for a small loop spanning the disk. It may or may not be useful to know that

$$\int_0^\infty dk\, J_1(k\rho)\, J_1(k\rho') = \frac{2\,[K(m) - E(m)]}{\pi\rho_<}, \qquad \text{where } m = \frac{\rho_<^2}{\rho_>^2}.$$

The remaining integral of the elliptic integral term over ρ' can be left as an integral, or expressed in hypergeometric functions.

4. A thin circular ring of radius a with charge $Q = 2\pi\lambda a$ uniformly distributed around its circumference lives in the x-y plane centered at the origin.

(a) Show that the electrostatic potential can be expressed using the elliptic integral as

$$\Phi = \frac{1}{4\pi\epsilon_0}\frac{4\lambda a\,K(m)}{(r^2 + a^2 + 2ar\sin\theta)^{1/2}}, \qquad m = \frac{4ar\sin\theta}{r^2 + a^2 + 2ar\sin\theta}$$

(note that the argument of K is positive, and that is a '+' sign in the denominator).

(b) What is the potential far from the ring? What is the potential near the center of the ring? What is the potential near the axis?

(c) Show that $\displaystyle\sum_{k=0}^{\infty}(-1)^k\,\frac{(2k-1)!!}{(2k)!!}\,P_{2k}(\cos\theta) = \frac{2}{\pi}\,\frac{K[2\sin\theta/(1+\sin\theta)]}{\sqrt{2(1+\sin\theta)}}$.

(d) What is the potential near the ring, that is, as $r \to a$ and $\theta \to \frac{\pi}{2}$? Comment.

4. A half-infinite solenoid consists of a cylinder of radius b centered on the negative z-axis. The cylinder carries an azimuthal surface current $\boldsymbol{K} = K\hat{\boldsymbol{\phi}}$.

(a) What is the magnetic field $\boldsymbol{B}$ everywhere along the z-axis?

(b) What is the vector potential near the z-axis? Compute $\nabla \times \boldsymbol{A}$ and compare with your answer in (a).

(c) What is the vector potential everywhere in space in the limit $b \to 0$, $K \to \infty$, such that the product $\pi b^2 K = g$ is fixed? Write your answer in spherical coordinates r, θ, ϕ (recall, $z = r\cos\theta$, and $\rho = r\sin\theta$). What is the resulting magnetic field $\boldsymbol{B}$? Comment.