8 | 24 | 2015

on ① due to ② from ① to ②

$$\boxed{\vec{F}_{12} = K \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}}$$

Coulomb. (*)

2 ± 0.04

$K \longleftrightarrow [q]$

S.I. $\quad K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{N\cdot m^2}{C^2} \quad (9 = 2.99792458^2)$

(± 0.001)

$$q' = \frac{q}{\sqrt{4\pi\varepsilon_0}} = \sqrt{K}\cdot q \qquad F = \frac{q_1' q_2'}{r^2}$$

Gaussian

(Errata: insert "$\frac{1}{4\pi\varepsilon_0}$" in eq. (x) .)

Force on q at $\vec{x}$ due to q_0 at $\vec{x}_0$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} q q_0 \frac{(\vec{x} - \vec{x}_0)}{|\vec{x} - \vec{x}_0|^3}$$

$\{q_1 \cdots q_N\}$: [linear superposition] (**)

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} q q_i \frac{(\vec{x} - \vec{x}_i)}{|\vec{x} - \vec{x}_i|^3} = q\vec{E}(\vec{x})$$

$$\vec{E}(\vec{x}) = \frac{1}{4\pi\varepsilon_0} \sum_i q_i \frac{(\vec{x}-\vec{x}_i)}{|\vec{x}-\vec{x}_i|^3}.$$

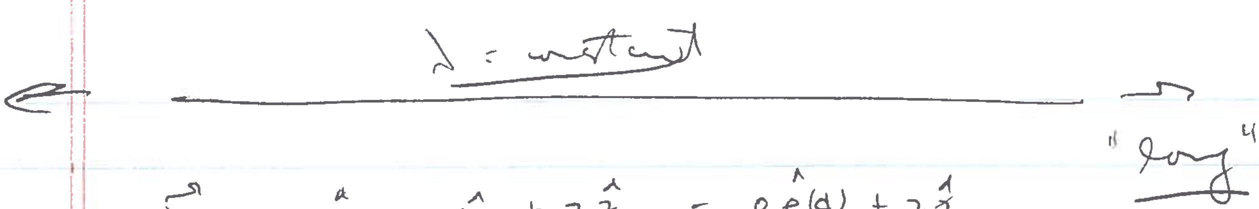
Continuous

$$\boxed{\vec{E}(\vec{x}) = \frac{1}{4\pi\varepsilon_0} \int \rho(\vec{x}')\, d^3x' \frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}} \quad (1.5)$$

$$|\vec{x}-\vec{x}'| = |\vec{\mathscr{r}}|$$

measure.

$$\int_V \rho\, d^3x' \to \int_S \sigma\, d^2a' \to \int_C \lambda\, dl' \to \sum_i q_i'$$



$$\vec{x} = x\hat{x} + y\hat{y} + z\hat{z} = \rho\,\hat{\rho}(\phi) + z\hat{z}$$

$$|\vec{x}-\vec{x}'|^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$$

$$= \rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi-\phi') + (z-z')^2$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \lambda\, dz' \frac{(\rho\hat{\rho} + z\hat{z}) - (\cancel{\rho'\hat{\rho}'} + z'\hat{z})}{\left(\rho^2 + \cancel{\rho'^2} - \cancel{2\rho\rho'\cos(\phi-\phi')} + (z-z')^2\right)^{3/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dz'\,\left(\rho\hat{\rho} + (z-z')\hat{z}\right)}{\left(\rho^2 + (z-z')^2\right)^{3/2}}$$

let $z'' = z' - z$ $\longrightarrow$ $\displaystyle\int dz' \frac{\rho\hat{\rho} - z''\hat{z}}{(\rho^2 + z''^2)^{3/2}}$ ~~odd~~

except direction

Field is radial (cylindrical), indep. of ϕ, z.

Symmetries: Invoke early

$$\vec{E} = \frac{\lambda\rho\hat{\rho}}{4\pi\varepsilon_0}\int_{-\pi/2}^{\pi/2} \frac{\rho\sec^2\theta\, d\theta}{\underbrace{(\rho^2\tan^2\theta + \rho^2)^{3/2}}_{\sec^3\theta}} = \int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = 2$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0}\frac{\hat{\rho}}{\rho}}$$