9/2/2015.

$\vec{F} = q\vec{E} = q(-\vec{\nabla}\Phi) = -\vec{\nabla}(q\Phi)$.

$\boxed{V = q\Phi}$ potential energy:
(charge) x (electrostatic potential)

$W = -\int \vec{F} \cdot d\vec{x} = -\int q\vec{E} \cdot d\vec{x} = \underline{q\Phi}$.

<u>point charge</u>. $\Phi_i = \frac{1}{4\pi\varepsilon_0} \frac{q}{|\vec{x}-\vec{x}_i|}$ $\left(\sum_i\right)$

<u>work to assemble constellation of charges</u>

1st/. ∵ no previous q. (free). $W_1 = 0$.

2nd/: $W_2 = \frac{1}{4\pi\varepsilon_0} \frac{q_2 q_1}{|\vec{x}_2 - \vec{x}_1|}$

3rd/. $W_3 = \frac{1}{4\pi\varepsilon_0} \frac{q_3 q_1}{|\vec{x}_3 - \vec{x}_1|} + \frac{1}{4\pi\varepsilon_0} \frac{q_3 q_2}{|\vec{x}_3 - \vec{x}_2|}$

<u>Nth</u>. $W_N = \frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{N-1} \frac{q_N q_j}{|\vec{x}_N - x_j|}$

<u>Total</u> $W = \sum_{i=1}^{N} W_i = \sum_{i=1}^{N} \sum_{j=1}^{N-1} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{|\vec{x}_i - \vec{x}_j|}$

(1.50)

②

to $j<i$ add $j>i$ $\boxed{W = \frac{1}{2} \sum_{i \neq j} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{|\vec{x}_i - \vec{x}_j|}}$ (1.51)

symmetric : independent of order.

Continuous:

$$\boxed{W = \frac{1}{8\pi\varepsilon_0} \int d^3x \, d^3x' \, \frac{\rho(\vec{x})\,\rho(\vec{x}')}{|\vec{x}-\vec{x}'|}} \quad (1.52)$$

$$= \frac{1}{2} \int d^3x \, \rho(\vec{x}) \left[\frac{1}{4\pi\varepsilon_0} \int d^3x' \, \frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} \right]$$

$$\boxed{W = \frac{1}{2} \int d^3x \, \rho(\vec{x}) \, \Phi(\vec{x})} \quad (1.53)$$

"reciprocity": (ρ in Φ') = (ρ' in Φ).

write $\rho = -\varepsilon_0 \nabla^2 \Phi$ $\quad W = \frac{1}{2} \int d^3x \left(-\varepsilon_0 \nabla^2 \Phi\right) \Phi$

$$= -\frac{1}{2}\varepsilon_0 \int d^3x \left[\vec{\nabla} \cdot \left(\Phi \vec{\nabla}\Phi\right) - |\vec{\nabla}\Phi|^2 \right]$$

$$= \underbrace{\frac{1}{2}\varepsilon_0 \oint d^2a' \, \hat{n}' \cdot \vec{E}\Phi}_{\substack{R \to \infty \\ (4\pi R^2)\left(\frac{Q}{R^2}\right)\left(\frac{Q}{R}\right) \to 0}} + \int d^3x \, \frac{1}{2}\varepsilon_0 |\vec{E}|^2$$

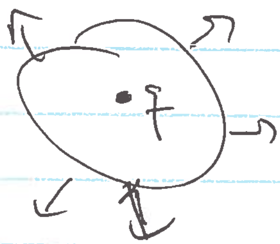
$$\boxed{W = \int d^3x \, \frac{1}{2}\varepsilon_0 |\vec{E}|^2} \quad (1.55)$$

$$\boxed{w = \frac{1}{2}\varepsilon_0 |\vec{E}|^2}$$

"Ideal conductor" →

charges free to move. so they will

$\vec{E} \to$ (force) → (readjustment until $\vec{E} \to 0$)
(dissipation).

(*).  $\oint \vec{E}\cdot\hat{n}\, d^2a = Q/\epsilon_0 = 0$

$\checkmark$ No charge in interior

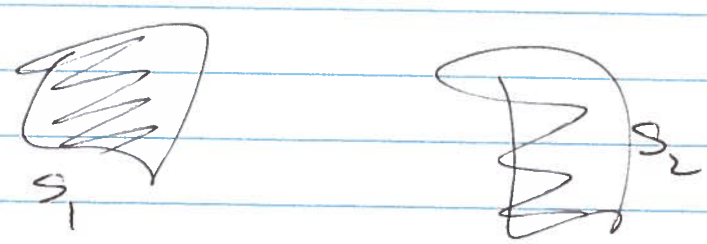
(*). $\Delta E_\parallel = 0$. $E_{\parallel,\text{inside}} = 0 \to \boxed{E_{\parallel,S} = 0}$ outside.

(*) $\Delta E_\perp = E_{\perp,S} - E_{\perp,\text{inside}} = \sigma/\epsilon_0$.

$\checkmark\ \boxed{\vec{E}|_S = \frac{\sigma}{\epsilon_0}\hat{n}}$ $\qquad E_{\perp,S} = \sigma/\epsilon_0$

(*) $\Delta\Phi_{\substack{\text{along} \\ \text{surface}}} = -\int \vec{E}\cdot d\vec{l} = -\int \vec{E}_\parallel\, dl. = 0$

$\checkmark$ Surface is Equipotential



System of conductors $S_1, S_2, \ldots$

conducting $\rightarrow$ $\Phi\big|_{S_i} = \text{constant} = V_i$

Φ is linear in sources: $\sum_i$

$$\boxed{V_i = \sum_j p_{ij} Q_j}$$ (real). symmetric $\longleftarrow$ invertible

$$\rightarrow \boxed{Q_i = \sum_j C_{ij} V_j} \quad (1.61).$$

diagonal $\rightarrow$ "capacities"   off diag. $\rightarrow$ "coefficients of induction"

$$W = \frac{1}{2}\int d^3x\, \rho(\vec{x})\,\Phi(\vec{x}) = \frac{1}{2}\sum_i \int_{S_i} d^2a\, \sigma_i(\vec{x})\, V_i$$

$$= \sum_i \frac{1}{2} Q_i V_i \qquad \boxed{W = \frac{1}{2} C_{ij} V_i V_j} \quad (1.62)$$

$$Q_i = C_{ij} V_j \rightarrow V_i = C^{-1}_{ij} Q_j$$

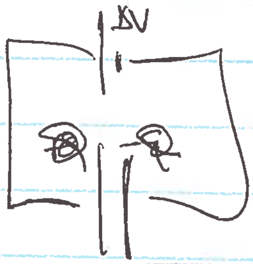
$$W = \frac{1}{2} C_{ij} \left(C^{-1}_{im} Q_m\right)\left(C^{-1}_{jn} Q_n\right) = \frac{1}{2} C^{-1}_{mi} C_{ij} C^{-1}_{jn} Q_m Q_n$$

$$W = \frac{1}{2} C^{-1}_{jk} Q_j Q_k$$

2 conductors

$$C_{ij} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

$$C^{-1}_{ij} = \frac{1}{C_{11}C_{22}-C_{12}C_{21}} \begin{pmatrix} C_{22} & -C_{12} \\ -C_{21} & C_{11} \end{pmatrix}.$$



$$\boxed{(\Delta V) = V_1 - V_2 = \frac{(C_{11}+C_{22}+C_{21}+C_{12})}{(C_{11}C_{22} - C_{12}C_{21})} Q} = Q/C$$

$$W = \frac{1}{2} \frac{(C_{11}+C_{22}+C_{12}+C_{21})}{(C_{11}C_{22}-C_{12}C_{21})} Q^2$$

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} C \cdot \Delta V^2.$$

ΔV, W unchanged. $V_i \to V_i + V_0$