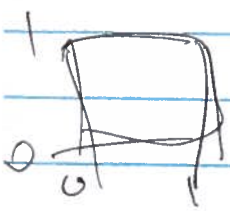
9/21/2015.

$$G(x,y;x',y') = \sum_{n=1}^{\infty} \frac{8}{n \sinh n\pi} \sin n\pi x \sin n\pi x' \sinh n\pi y_< \sinh n\pi(1-y_>).$$



$$\nabla^2 G = \nabla'^2 G = -4\pi \delta^{(2)}(\vec{x}-\vec{x}') = 0 \quad A\vec{x}.$$

$$G\big|_{x=0} = G\big|_{x'=0} = G\big|_{x=1} = G\big|_{x'=0} = G\big|_{y_<=0} = G\big|_{y_>=1} = 0.$$

$$G(\vec{x},\vec{x}') = G(\vec{x}',\vec{x}) \Leftarrow$$ can use to write form easily.

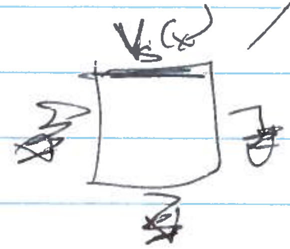
$$\frac{\sinh n\pi y_< \sinh n\pi(1-y_>)}{\sinh n\pi} \rightarrow \frac{\frac{1}{2}e^{n\pi y_<} \cdot \cancel{\tfrac{1}{2}} e^{n\pi(1-y_>)}}{\cancel{\tfrac{1}{2}} e^{n\pi}}$$

n large

$$= \left(\tfrac{1}{2}\right) e^{-n\pi(y_> - y_<)} = e^{-n\pi|y-y'|}$$

(2)

$$\Phi(\vec{x}) = \int d^3x' \, G\rho - \frac{1}{4\pi}\int da' \, \Phi\big|_S \frac{\partial G}{\partial n'}$$



top: $\hat{n} = \hat{y}$ $\quad \frac{\partial G}{\partial n'} = \frac{\partial G}{\partial y'}$

$$= \sum_{n=1}^{\infty} \frac{8}{\sinh n\pi} \cdot \sin n\pi x \cdot \sin n\pi x' \cdot \underset{y_< = y}{\sinh n\pi y} \cdot \cancel{\left(-n\pi \cosh n\pi(1-y)\right)}$$

(2)

$$\Phi(x,y) = \frac{+1}{\cancel{4\pi}}\int dx' \sum_{n=1}^{\infty} \frac{8}{n\cancel{\sinh n\pi}} \sin n\pi x \sin n\pi x' \sinh n\pi y \, V_S(x') \, \cancel{(4\pi)}$$

$$= \sum_{n=1}^{\infty} \left(2\int_0^1 dx' \sin n\pi x' \, V_S(x')\right) \sin n\pi x \frac{\sinh n\pi y}{\sinh n\pi}$$

$V_S = 1$. $\Phi_c =$

0.253716

0.249904

0.2500029 (3 0's)

0.24999900 (4 9's)

0.2500000034 (6 0's)

0.24999999987 (7 9's)

0.2500000000047 (9 0's).

③

$$f(x \pm h) = f(x) \pm h f'(x) + \frac{1}{2} f''(x) \cdot h^2 + \dots$$

$$f(x+h) + f(x-h) - 2f(x) = f''(x) \cdot h^2 + O(h^4).$$

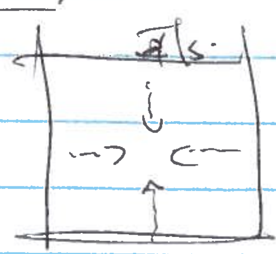
$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + O(h^2)$$

(3D). $\nabla^2 \Phi = \Big[\Phi(x+h) - 2\Phi(x) + \Phi(x-h)$
$+ \Phi(y+h) - 2\Phi(y) + \Phi(y-h)$
$+ \Phi(z+h) - 2\Phi(z) + \Phi(z-h) \Big] / h^2$

$\nabla^2 \Phi = 0 \rightarrow \Phi(x+h) + \Phi(x-h) + \Phi(y+h) + \Phi(y-h)$
$+ \Phi(z+h) + \Phi(z-h) = 6\Phi(x,y,z)$

$$\Phi(x,y,z) = \frac{1}{6} \Big[\Phi(x+h) + \Phi(x-h) + \Phi(y+h) + \Phi(y-h) + \Phi(z+h) + \Phi(z-h) \Big]$$

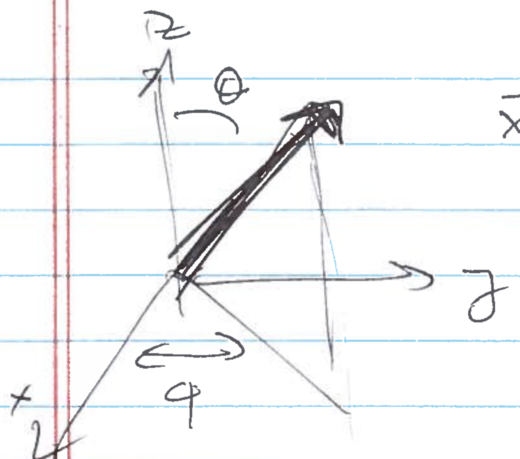
Numerical Solution



fix Φ_s, iterate.

fine grid: accurate, slow.
N^2

multigrid order N ?



$$\vec{x} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$\hat{r} = \frac{\partial\vec{x}}{\partial r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

$$\hat{\theta} = \frac{1}{r}\frac{\partial\vec{x}}{\partial\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$$

$$\hat{\phi} = \frac{1}{r\sin\theta}\frac{\partial\vec{x}}{\partial\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

$$\frac{\partial\hat{r}}{\partial\theta} = \hat{\theta} \qquad \frac{\partial\hat{\theta}}{\partial\theta} = -\hat{r} \qquad \frac{\partial\hat{\phi}}{\partial\theta} = 0$$

$$\frac{\partial\hat{r}}{\partial\phi} = \sin\theta\,\hat{\phi} \qquad \frac{\partial\hat{\theta}}{\partial\phi} = \cos\theta\,\hat{\phi} \qquad \frac{\partial\hat{\phi}}{\partial\phi} = -\hat{r}\sin\theta - \hat{\theta}\cos\theta = -\hat{\rho}$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial}{\partial\theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial}{\partial\phi}.$$

$$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$$

5

$$\vec{\nabla}\cdot\vec{A} = \hat{r}\cdot\left(\hat{r}\frac{\partial A_r}{\partial r} + A_r\cdot\cancel{\frac{\partial \hat{r}}{\partial r}} + \frac{\hat{\theta}}{r}\frac{\partial A_\theta}{\partial r} + \cancel{A_\theta\frac{\partial \hat{\theta}}{\partial r}} \cdots\right)$$

$$+ \frac{\hat{\theta}}{r}\cdot\left(\cancel{\hat{r}\frac{\partial A_r}{\partial \theta}} + A_r\frac{\partial \hat{r}}{\partial \theta} + \hat{\theta}\frac{\partial A_\theta}{\partial \theta} + \cdots\right)$$

$\hat{\theta}$

$$+ \frac{\hat{\phi}}{r\sin\theta}\cdot\left(\cdots + A_r\frac{\partial \hat{r}}{\partial \phi} + \cdots + A_\theta\frac{\partial \hat{\theta}}{\partial \phi} + \hat{\phi}\frac{\partial A_\phi}{\partial \phi}\right)$$

$\sin\theta\,\hat{\phi}$ $\quad$ $\cos\theta\,\hat{\phi}$

$$\vec{\nabla}\cdot\vec{A} = \left(\frac{\partial A_r}{\partial r} + \frac{2A_r}{r}\right) + \frac{1}{r}\left(\frac{\partial A_\theta}{\partial \theta} + \frac{\cos\theta}{\sin\theta}A_\theta\right)$$

$$+ \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

(Back cover)
(on web)

$\vec{A} = \vec{\nabla}\Phi$

$$\vec{\nabla}\cdot(\vec{\nabla}\Phi) = \nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \Phi}{\partial \theta}\right)$$

$$+ \frac{1}{r^2}\frac{1}{\sin^2\theta}\frac{\partial^2 \Phi}{\partial \phi^2}$$

Separation: $\Phi = R(r)\, P(\theta)\, Q(\phi)$.

$$\frac{r^2}{\Phi}\nabla^2\Phi = \frac{r^2}{R}\left(\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr}\right) + \frac{1}{P\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right) + \frac{1}{\sin^2\theta}\cdot\frac{1}{Q}\frac{d^2Q}{d\phi^2} = 0.$$

Take: $\frac{1}{Q}\frac{d^2Q}{d\phi^2} = \text{constant} = -m^2$ $\quad Q = e^{im\phi}$

periodic $\rightarrow$ m = integer. (+/- values)

Always $m=0$ $\quad Q = Q_0 + \cancel{Q_1\phi}$ not periodic.

$$\frac{1}{P\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dP}{d\theta}\right) - \frac{m^2}{\sin^2\theta} = \text{constant} = -\ell(\ell+1)$$

$-$ oscillates.

$$\frac{r^2}{R}\left(\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr}\right) = +\ell(\ell+1).$$

$$r^2R'' + 2rR' - \ell(\ell+1)R = 0$$

$R \sim r^p$ $\quad p(p-1) + 2p - \ell(\ell+1) = 0$

$p(p+1) = \ell(\ell+1)$ $\quad p = \ell$ $\quad p = -(\ell+1)$ $\quad \boxed{R \sim r^\ell, \frac{1}{r^{\ell+1}}}$