10/12/2015 $\boxed{J_m(x_{mn}) = 0}$

$$\int_0^a \rho\, d\rho\, J_m\left(\frac{x'\rho}{a}\right)\left\{\frac{1}{\rho}\frac{d}{d\rho}\left(\rho \frac{dJ_m}{d\rho}\left(\frac{x\rho}{a}\right)\right) + \left(\frac{x^2}{a^2} - \frac{m^2}{\rho^2}\right) J_m\left(\frac{x\rho}{a}\right)\right\} = 0$$

parts.

symmetric

$$\left[J_m\left(\frac{x'\rho}{a}\right)\rho \frac{dJ_m}{d\rho}\left(\frac{x\rho}{a}\right)\right]_0^a - \int_0^a \rho\, d\rho \left[\frac{dJ_m}{d\rho}\left(\frac{x'\rho}{a}\right)\rho\frac{dJ_m}{d\rho}\left(\frac{x\rho}{a}\right)\right]$$

$$+ \int_0^a \rho\, d\rho \left(\frac{x^2}{a^2} - \frac{m^2}{\rho^2}\right) J_m\left(\frac{x'\rho}{a}\right) J_m\left(\frac{x\rho}{a}\right) = 0$$

$x \to x'$, subtract

$$\left[J_m\left(\frac{x'\rho}{a}\right)\rho \frac{dJ_m}{d\rho}\left(\frac{x\rho}{a}\right)\right]_{\rho=a} - \left[J_m\left(\frac{x\rho}{a}\right)\rho\frac{dJ_m}{d\rho}\left(\frac{x'\rho}{a}\right)\right]$$

$$+ \left(\frac{x^2 - x'^2}{a^2}\right)\int_0^a \rho\, d\rho\, J_m\left(x'\frac{\rho}{a}\right) J_m\left(\frac{x\rho}{a}\right) = 0$$

Last time $x = x_{mn}$, $x' = x_{mn'}$ $\to$ $\delta_{nn'}$

Now: $x' = x_{mn} + \varepsilon$ $\quad x = x_{mn}$.

②

$$J_m(x_{mn}+\varepsilon) \cdot \left(\frac{x_{mn}}{a}\right) J_m'(x_{mn})$$

$$= \frac{\cancel{(x^2} + 2\varepsilon x + \cancel{\varepsilon^2)} - \cancel{x^2}}{a^2} \int_0^a \rho\, d\rho\, J_m^2\left(\frac{x_{mn}\rho}{a}\right)$$

$$x_{mn} J_m'(x_{mn}) \left[\cancel{J_m(x_{mn})} + \varepsilon J_m'(x_{mn})\right]$$

$$= \frac{2\varepsilon x_{mn}}{a^2} \int_0^a \rho\, d\rho\, J_m^2\left(\frac{x_{mn}\rho}{a}\right)$$

$$\boxed{\int_0^a \rho\, d\rho\, J_m^2\left(\frac{x_{mn}\rho}{a}\right) = \frac{1}{2} a^2 \left[J_m'(x_{mn})\right]^2}$$

$-$ $\quad J_{m-1} - J_{m+1} = 2 J_m'$

$+$ $\quad J_{m-1} + J_{m+1} = \frac{2m}{x} J_m$

$$2 J_{m+1} = \frac{2m}{x} J_m - 2 J_m'$$

$$J_m'(x_{mn}) = -J_{m+1}(x_{mn})$$

$$\boxed{\int_0^a \rho\, d\rho\, J_m^2\left(\frac{x_{mn}\rho}{a}\right) = \frac{1}{2} a^2 \left[J_{m+1}(x_{mn})\right]^2}$$

$$\Phi = \sum_{m'=-\infty}^{\infty} \sum_{n'=1}^{\infty} A_{m'n'} \cdot J_{m'}\left(\frac{x_{m'n'}\rho}{a}\right) e^{im'\phi} \sinh k_{m'n'} z$$

Case: $V_s(\rho,\phi) = V_0$ = constant.

multiply both sides $e^{-im\phi}$, $\int_0^{2\pi} d\phi \rightarrow 2\pi\, \delta_{mm'}$

multiply both $J_m\left(\frac{x_{mn}\rho}{a}\right)$, $\int_0^a \rho\, d\rho \rightarrow \frac{1}{2} a^2 \left[J_{m+1}(x_{mn})\right]^2 \delta_{nn'}$

$$\sum_{m'=-\infty}^{\infty} \sum_{n'=1}^{\infty} A_{m'n'} \cdot 2\pi\, \delta_{mm'} \cdot \frac{1}{2} a^2 \left[J_{m+1}(x_{mn})\right]^2 \delta_{nn'} \sinh\left(\frac{x_{mn}L}{a}\right)$$

$$= \int_0^{2\pi} d\phi \int_0^a \rho\, d\rho\, \underbrace{e^{-im\phi}}_{2\pi\,\delta_{m,0}} J_m\left(\frac{x_{mn}\rho}{a}\right) V_0$$

$$\int_0^a \rho\, d\rho\, J_0\left(x_{0n}\frac{\rho}{a}\right) = \frac{a^2}{x^2} \int_0^x y\, dy\, J_0(y)$$

$$J_0 - J_2 = 2\frac{dJ_1}{dx}$$
$$J_0 + J_2 = \frac{2}{x} J_1$$

$$J_0 = \frac{1}{y} J_1 + J_1'$$
$$y J_0 = J_1 + y J_1' = \frac{d}{dy}(y J_1)$$

$$\rightarrow \frac{a^2}{x^2} \int_0^x \frac{d}{dy}(y J_1) = \frac{a^2}{x^2}\left[\cancel{y} J_1\right]_{\cancel{0}}^x = \frac{a^2 J_1}{x}$$

$$A_{0n} \cdot \tfrac{1}{2}\cancel{a}^2 \left[J_1(x_{0n})\right]^2 \cdot \cancel{2\pi} \cdot \sinh\left(\frac{x_{0n} z}{L}\right)$$

$$= \cancel{a}^2 \frac{J_1(x_{0n})}{x_{0n}} \cdot \cancel{2\pi} \cdot V_0$$

$$A_{0n} = \frac{2}{x_{0n} J_1(x_{0n})} \frac{1}{\sinh\left(\frac{x_{0n} L}{a}\right)}$$

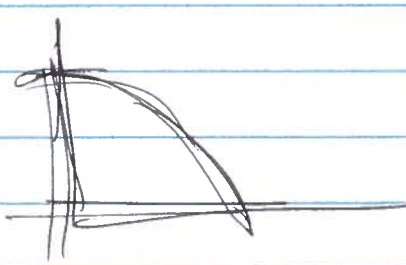
$$\boxed{\Phi = \sum_{n=1}^{\infty} \frac{2 V_0}{x_{0n} J_1(x_{0n})} J_0\left(\frac{x_{0n}\rho}{a}\right) \cdot \frac{\sinh\left(x_{0n} z/a\right)}{\sinh\left(x_{01} L/a\right)}}$$

$L \gg a$ $$\frac{\sinh\left(\frac{xz}{a}\right)}{\sinh\left(\frac{xL}{a}\right)} \simeq \frac{\frac{1}{2} e^{xz/a}}{\frac{1}{2} e^{xL/a}} = e^{-x_{0n}\left(\frac{L-z}{a}\right)}$$

smallest . $x_{01} = 2.405$ dominates

$\left(n - \frac{1}{4}\right)\pi$

(except very near top).



$L \ll a$ $$\frac{\sinh\left(\frac{xz}{a}\right)}{\sinh\left(\frac{xL}{a}\right)} \simeq \frac{\frac{xz}{a}}{\frac{xL}{a}} = \frac{z}{L} \qquad V_0(\rho, \varphi) \times \left(\frac{z}{L}\right)$$