10/16/2015

$$\frac{1}{|\vec{x}-\vec{x}'|} = \sum_{m=-\infty}^{\infty} \int_0^{\infty} dk\, J_m(k\rho) J_m(k\rho') e^{im(\phi-\phi')} e^{-k|z-z'|}$$

$$J_0\left(k\sqrt{\rho^2+\rho'^2-2\rho\rho'\cos(\phi-\phi')}\right) = \sum_{m=-\infty}^{\infty} J_m(k\rho) J_m(k\rho') e^{im(\phi-\phi')}$$

let ($\phi' \to 0$) $\rho' \to$ large.

$$\sqrt{\rho^2+\rho'^2-2\rho\rho'\cos(\phi-\phi')} \approx \rho' - \rho\cos\phi + \cancel{O\left(\frac{\rho^2}{\rho'}\right)}$$

$$J_0\left(k(\rho'-\rho\cos\phi)\right) = \underline{\sqrt{\frac{2}{\pi k\rho'}}\cos\left(k\rho' - k\rho - \frac{\pi}{4}\right)}$$

$$= \underline{\sum_{m=-\infty}^{\infty} J_m(k\rho) e^{im\phi} \sqrt{\frac{2}{\pi k\rho'}}\cos\left(k\rho' - \frac{m\pi}{2} - \frac{\pi}{4}\right)}$$

2

$$\cos(k\rho\cos\phi) = \sum_{m=-\infty}^{\infty} J_m(k\rho)\cos\left(\frac{m\pi}{2}\right) e^{im\phi}$$

$$\sin(k\rho\cos\phi) = \sum_{m=-\infty}^{\infty} J_m(k\rho)\sin\frac{m\pi}{2} e^{im\phi}.$$

$$(*)\qquad e^{ik\rho\cos\phi} = \sum_{m=-\infty}^{\infty} J_m(k\rho)\, e^{i\frac{m\pi}{2}} e^{im\phi}$$

$$\phi=0\qquad \mathrm{Re}\left[\sum_{m=-\infty}^{\infty} i^m J_m(x)\right] = \cos x.$$

$$= \sum_{k=0}^{\infty}(-1)^k J_{2k}(x) = J_0 + 2\sum_{k=1}^{\infty}(-1)^k J_2k(x)$$

$$\mathrm{Im}\left[\sum_{m=-\infty}^{\infty} i^m J_m(x)\right]$$

$$= 2\sum_{k=0}^{\infty}(-1)^{k+1} J_{2k+1}(x) = \sin x$$

$$(*)\qquad \int_0^{2\pi}\frac{d\phi}{2\pi} e^{-im\phi}\left[\sum_{m'=-\infty}^{\infty} i^{m'} J_{m'}(x)\, e^{im'\phi}\right]$$

$$= i^m J_m(x). = \int_0^{2\pi}\frac{d\phi}{2\pi} e^{ix\cos\phi - im\phi}$$

③

$m=0$ stationary points. $\phi = 0 \qquad \phi \simeq \pi$

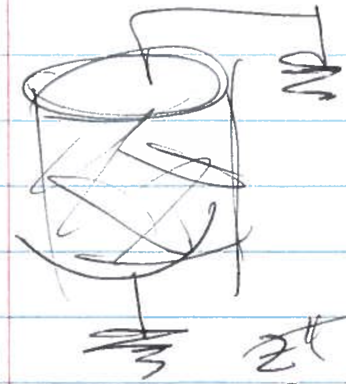
$$\int \frac{d\phi}{2\pi} e^{ix\cos\phi} \simeq \int \frac{d\phi}{2\pi} e^{ix\left(1-\frac{1}{2}\phi^2\right)} + \int \frac{d\phi'}{2\pi} e^{ix\left(-1+\frac{1}{2}\phi'^2\right)}$$

$$= \frac{1}{2\pi} \cdot e^{ix} \underline{\int d\phi\, e^{-\frac{1}{2}ix\phi^2}} + \text{c.c.}$$

$$= \frac{1}{2\pi} \cdot e^{ix} \cdot \sqrt{2\pi \frac{1}{ix}} + \text{c.c.}$$

$$= \frac{1}{\sqrt{2\pi x}} e^{ix} e^{-i\frac{\pi}{4}} + \text{c.c.}$$

$$\boxed{J_0(x) \sim \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4}\right)}$$



$\Phi(\text{top}) = \Phi(\text{bottom}) = 0.$

$\Phi|_S = V_S(\phi, z)$ on sides.

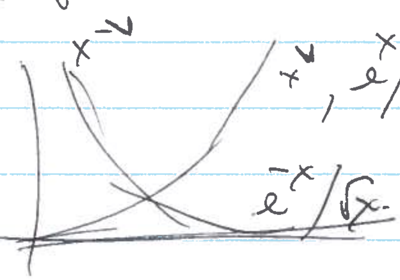
$\frac{Z''}{Z} = +k^2 \rightarrow e^{\pm ikz} \rightarrow \sin kz$

$\rightarrow \sin\left(\frac{n\pi z}{L}\right)$

$$\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} + \left(-k^2 - \frac{m^2}{\rho^2}\right)R = 0.$$

$$\frac{d^2F}{dx^2} + \frac{1}{x}\frac{dF}{dx} - \left(1 + \frac{\nu^2}{x^2}\right)F = 0.$$

positive feedback: convex up, no zeros.



$x \rightarrow ix$ every term but one changes sign

$a_{n+2} = (+) \cdot a_n$. all $\oplus$ signs, $\begin{matrix}\sin \\ \cos\end{matrix} \rightarrow \begin{matrix}\sinh \\ \cosh\end{matrix}$

"modified" $I_\nu = i^{-\nu} J_\nu(ix) \rightarrow \frac{e^{x}}{\sqrt{2\pi x}}$ (3.100)

$K_\nu = i^{\nu+1}\frac{\pi}{2} H_\nu^{(1)}(ix) \rightarrow \frac{\pi}{2}\sqrt{\frac{2}{\pi x}}\, e^{i(ix)} = \sqrt{\frac{\pi}{2x}}\, e^{-x}$

<u>many types of Bessel series.</u>

$$\sum_{n=1}^{\infty} a_n J_m(x_{mn} x) \qquad f(x)=0 \text{ @ } x=1.$$

$$\sum_{n=1}^{\infty} a_n J_{\nu+n}(x). \qquad (?)$$

$$\sum_{n=0}^{\infty} a_n J_{\nu+n}((\nu+n)x). \qquad \text{"Kapteyn" series}$$

$$\frac{1}{1-x\cos\psi} = 1 + 2\sum_{n=1}^{\infty} J_n(nx)\cos n\phi, \qquad \phi = \psi + x\sin\psi$$

$$\frac{1}{1-z} = 1 + 2\sum_{n=1}^{\infty} J_n(nz)$$

planetary dynamics.

$$E = M + \sum_{n=1}^{\infty} \frac{2}{n} J_n(ne)\sin(nM)$$

mean anomaly

$$M = E - e\sin E$$

$$r = \frac{a(1-e^2)}{1+e\cos\theta} = a(1-e\cos E)$$

true anomaly   eccentric anomaly

$$\sum_{n=0}^{\infty} a_n J_0(nx) \qquad \sum_{n \text{ odd}} \frac{J_0(nx)}{n^2} = \frac{1}{2}\left(\frac{\pi^2}{4} - 1\right) \ldots \frac{x^2}{4}$$

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n J_0(nx) \qquad \text{"Schlömilch's series"}$$

Problem 14.3

$$\frac{dP_m}{d\Omega} = \frac{e^2 v^4}{2\pi a^2 c^3}\, m^2 \left[J_m'^2\!\left(\frac{mv\sin\theta}{c}\right) + \frac{\cot^2\theta}{v^2/c^2} J_m^2\!\left(\frac{mv}{c}\sin\theta\right)\right]$$