10/21/2015

$$\Phi = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r} + \frac{\vec{p}\cdot\vec{x}}{r^3} + \frac{1}{6}Q_{ij}\frac{x_i x_j}{r^5} + \cdots\right)$$

$$= \frac{1}{4\pi\epsilon_0}\sum_{l,m}\frac{4\pi}{2l+1}\frac{Y_{lm}}{r^{l+1}} q_{lm}$$

Ex.

$$\langle\vec{E}(\vec{x})\rangle_R = \frac{1}{\frac{4\pi}{3}R^3}\int_0^R d^3x\,(-\vec{\nabla}\Phi)$$

$$= \frac{1}{V}\oint_{r=R} R^2 d\Omega\,(-\hat{r}\,\Phi)$$

$$= -\frac{1}{V}\int R^2 d\Omega\,\hat{r}\,\frac{1}{4\pi\epsilon_0}\int d^3x'\,\frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|}$$

$$= -\frac{1}{V}\frac{R^2}{4\pi\epsilon_0}\int d^3x'\,\rho(\vec{x}')\int d\Omega\,\frac{\hat{r}}{|\vec{x}-\vec{x}'|}$$

$$\int d\Omega\, \hat{r} \left(\frac{1}{r} + \frac{\hat{r}\cdot\hat{r}'\, r_<}{r_>^2} + \frac{P_2(\hat{r}\cdot\hat{r}')\, r_<^2}{r_>^3} + \cdots \right)$$

$\ell = 1$

$$\int d\Omega \frac{\hat{r}}{|\vec{x}-\vec{x}'|} = \int d\Omega\, \hat{r}\,(\hat{r}\cdot\hat{r}')\frac{r_<}{r_>^2} = \int d\Omega\, \hat{r}'_j (\hat{r}_i \hat{r}_j)$$

$$\int d\Omega\, \hat{r}_i \hat{r}_j = C \cdot \delta_{ij} \qquad \int d\Omega \cdot \cos^2\theta = 2\pi \int_{-1}^{1} dx \cdot x^2 = \frac{4\pi}{3} = C$$

$$\int d\Omega\, \hat{r}_i \hat{r}_j = \frac{4\pi}{3}\delta_{ij}$$

$$\boxed{\int d\Omega \frac{\hat{r}}{|\vec{x}-\vec{x}'|} = \frac{4\pi}{3}\frac{r_<}{r_>^2}\cdot \hat{r}'}$$

$$\langle \vec{E}(\vec{x}) \rangle_R = \frac{1}{\frac{4\pi}{3}R^3}\Big[-\cancel{R^2} \int_0^R d^3x' \frac{1}{4\pi\epsilon_0} \cdot \frac{r'}{\cancel{R^2}} \hat{r}' \rho(\vec{x}') \cdot \frac{4\pi}{3}$$

$$- R^2 \int_R^\infty d^3x' \frac{1}{4\pi\epsilon_0} \frac{R}{r'^2} \hat{r}' \rho(\vec{x}') \frac{4\pi}{3} \Big)$$

$$= \frac{1}{\left(\frac{4\pi}{3}R^3\right)}\left(\frac{-1}{3\epsilon_0}\right)$$

3

$$\langle \vec{E} \rangle = \frac{1}{\frac{4\pi}{3}R^3}\left(-\frac{1}{3\varepsilon_0}\int_0^R d^3x' \cdot \vec{x}'\, \rho(\vec{x}') \right.$$

$$\left. + \cancel{\left(\frac{1}{\frac{4\pi}{3}R^3}\right)}\cancel{\left(\frac{4\pi}{3}R^3\right)}\int_R^\infty d^3x'\, \rho(\vec{x}')\left(\frac{0-\hat{r}'}{|0-\vec{x}'|^2}\right)\right)$$

$$\frac{\vec{0}-\vec{x}'}{|0-\vec{x}'|^3}$$

$$\boxed{\langle \vec{E}(\vec{x})\rangle_R = -\frac{1}{3\varepsilon_0}\frac{\vec{p}_{\text{inside}}}{V} + \vec{E}_{\text{outside}}(0)}$$

(4.18) (4.19)

point dipole. $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{3\hat{r}(\hat{r}\cdot\vec{p})-\vec{p}}{r^3} + \vec{K}\cdot\delta(\vec{x}')$

$$\int d^3x \cdot \vec{E} = \int d^3x \frac{1}{4\pi\varepsilon_0}\frac{3\hat{r}(\hat{r}\cdot\vec{p})-\vec{p}}{r^3} + \vec{K}$$

$$\int d\Omega \left(3\hat{r}_i\hat{r}_j p_j - p_i\right) = 3\cdot\frac{4\pi}{3}\delta_{ij}\,\hat{r}_i\hat{r}_j p_j - p_i$$

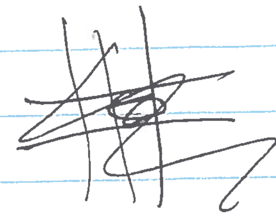
$$= \left(-\frac{1}{3\varepsilon_0}\right)\cdot\vec{p} \qquad \vec{K} = -\frac{1}{3\varepsilon_0}\vec{p}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{3\hat{r}(\hat{r}\cdot\vec{p})-\vec{p}}{r^3} - \frac{1}{3\varepsilon_0}\vec{p}\,\delta^{(3)}(\vec{x})}$$

$0, 0, \frac{1}{r}$

$$= -\frac{4\pi}{3}\delta_{ij}\,\delta(\vec{x})$$

Stuff .



Each little volume has $\underline{Q}$, $\vec{P}$, Q_{ij}

ρ = local density of Q.

$\vec{P}$ = local density of $\vec{P}$

higher order: { harder to order.
falls off faster. → ignore
doesn't contribute to $\langle \vec{E} \rangle$

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\left(\sum \frac{Q}{r} + \sum \frac{\vec{P}\cdot\vec{x}}{r^3} + \cancel{\cdots} \right)$$

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int d^3x' \left[\frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} + \vec{P}(\vec{x}')\cdot\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3} \right]$$

know: $\vec{\nabla}' \frac{1}{|\vec{x}-\vec{x}'|} = \frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}$

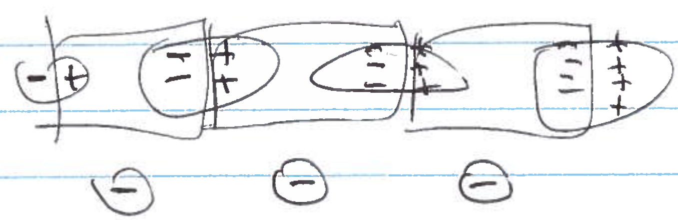
$$\vec{\nabla}'\cdot\left(\frac{\vec{P}(\vec{x}')}{|\vec{x}-\vec{x}'|}\right) = \frac{(\vec{\nabla}'\cdot\vec{P}(\vec{x}'))}{|\vec{x}-\vec{x}'|} + \vec{P}\cdot\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}$$

$$\int d^3x' \, \vec{P}(\vec{x}') \cdot \frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3} = \int d^3x' \left[\underbrace{\vec{\nabla}' \cdot (\ldots)} - \frac{(\vec{\nabla}' \cdot \vec{P}(\vec{x}'))}{|\vec{x}-\vec{x}'|} \right]$$

$$\rightarrow \oint_{R \to \infty} d^2a \, \hat{n} \cdot (\;) \rightarrow 0$$

$$\boxed{\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0} \int d^3x' \frac{[\rho(\vec{x}') - \vec{\nabla}' \cdot \vec{P}(\vec{x}')]}{|\vec{x}-\vec{x}'|}}$$

"ρ_P" $= -\vec{\nabla} \cdot \vec{P}$ $\qquad$ $\boxed{\sigma_P = \hat{n} \cdot \vec{P}}$



$\boxed{\vec{E} = -\vec{\nabla}\Phi}$ $\qquad$ $\boxed{\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0}(\rho - \vec{\nabla} \cdot \vec{P})}$

Let $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ $\qquad$ "displacement"

$\boxed{\vec{\nabla} \cdot \vec{D} = \rho}$ $\qquad$ local excess charge

"free charge"

$$\Delta E_\perp = \frac{\sigma_{tot}}{\varepsilon_0} = \Delta D_\perp - \Delta P_\perp = \sigma_{free} + \sigma_P$$

$\boxed{\sigma_P = -\Delta P_\perp}$ $\Rightarrow$ $\vec{P}_{inside}$ to 0 outside

$\boxed{\sigma_P = \hat{n} \cdot \vec{P}}$