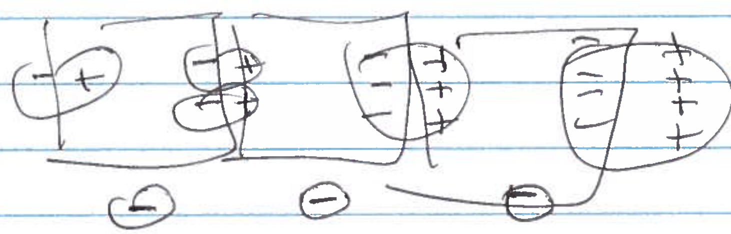
10/23/2015

$$\Phi = \frac{1}{4\pi\varepsilon_0}\int d^3x' \left[\frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} + \frac{\vec{P}(\vec{x}')\cdot(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3} \right]$$

$$\vec{E} = -\vec{\nabla}\Phi \qquad \vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\left(\rho - \vec{\nabla}\cdot\vec{P}\right)$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \qquad \vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\vec{\nabla}\cdot\vec{P} = -\rho_P$$



$$\Delta E_\perp = \frac{1}{\varepsilon_0}\left(\Delta D_\perp - \Delta P_\perp\right) \qquad \sigma_f = \Delta D_\perp$$

$$\sigma_P = -\Delta P_\perp$$

$$\boxed{\vec{P}} \quad 0 \qquad \vec{\nabla}\cdot\vec{P} = -\vec{P}_0\,\delta(\text{surface})\cdot\hat{n}$$

$$\sigma_P = \vec{P}\cdot\hat{n}$$

Why is there a $\vec{P}$?

spontaneous , induced .

$\vec{P}_0$ "Hard" , stretch, align by external field .

$\vec{P} = \vec{P}(\vec{E})$ "Constitutive relation" (equation of state)

local $\vec{P}$ at $\vec{x}$ $\leftrightarrow$ $\vec{E}$ at $\vec{x}$ (not $\int d^3x'$).

linear $P_i = \varepsilon_0 \chi_{ij} E_j$ $\quad \vec{P} = \varepsilon_0 (\overleftrightarrow{\chi}) \vec{E}$

principal axes . (eigenvalues) $\begin{pmatrix} \chi_1 & & \\ & \chi_2 & \\ & & \chi_3 \end{pmatrix}$

isotropic $\chi_1 = \chi_2 = \chi_3$. $\quad \vec{P} = \varepsilon_0 \chi) \vec{E}$

$$\vec{D} = \varepsilon_0 \vec{E} + (\varepsilon_0 \chi \vec{E}) = \varepsilon_0 (1+\chi) \vec{E} = \varepsilon_0 K \vec{E}$$

Dielectric Constant

$\varepsilon = \varepsilon_0 K$ permittivity

K = "relative permittivity" (dimensionless

Orientation

$$\langle p_z \rangle = \frac{\int d\Omega \cdot p\cos\theta \cdot e^{\beta p E\cos\theta}}{\int d\Omega \, e^{\beta p E \cos\theta}}$$

$$= p\left[\coth \beta p E - \frac{1}{\beta p E}\right] \simeq \frac{1}{3}\beta p^2 E$$

$$P = n\langle p \rangle = \frac{1}{3} n \frac{p^2 \cancel{E}}{kT} = \varepsilon_0 \chi \cancel{E}$$

$$\chi = \frac{1}{3}\frac{n p^2}{\varepsilon_0 kT} = 12 \quad (80)$$

$$n = \frac{1}{18}\cdot\left(6\times10^{23}\ cm^{-3}\right) = \frac{1}{6}\times10^{29}\ m^{-3} = 3\times10^{28}\ m^{-3}$$

$$3.344\times10^{28}\ m^{-3}$$

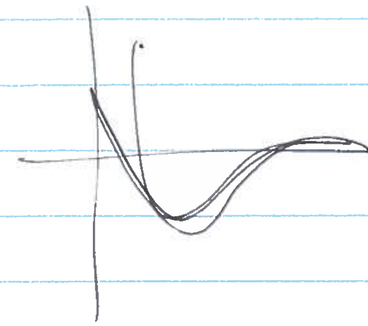
$$\frac{1}{\varepsilon_0} = 4\pi \times 9\times10^{9}$$

$$p = 1.85\ D = 1.85\left(3.3356\times10^{-30}\ C\cdot m\right) = 6.17\times10^{-30}\ C\cdot m$$

Debye $= 10^{-10}$ esu ~~cm~~

$$kT = \frac{1}{40}\ eV. = 4\times10^{-21}\ J.$$

Sketch



$F = -m\omega_0^2 x = eE$

$x = \frac{eE}{m\omega_0^2}$ $\qquad p = ex.$

$P = n \cdot \frac{e^2}{m\omega_0^2} \cdot E = \varepsilon_0 \chi E$

$$\chi = \frac{ne^2}{\varepsilon_0 m \omega_0^2} = \frac{(10^{29})(1.6\times10^{-19})^2}{(8.85\times10^{-12})(9.11\times10^{-31})(4\times10^{15})^2}$$

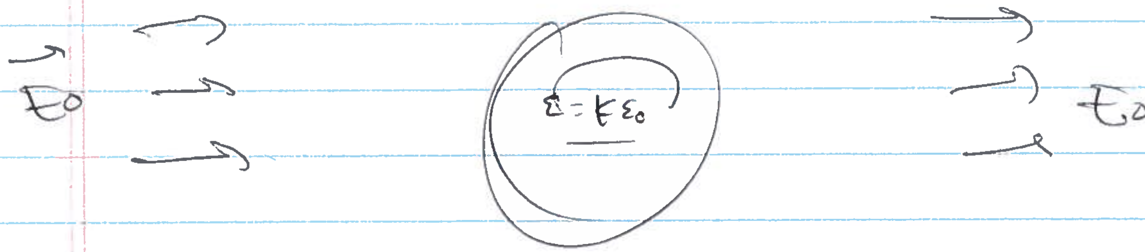
$\sim$ few.

QM. $\delta H = e\Phi = -e\vec{E}\vec{x}$

$\langle p\rangle = \langle nlm | e\vec{x} | nlm\rangle$ $\quad \to$ same

$p = \gamma E$ $\qquad \gamma = \gamma_a + \frac{\beta p_0^2}{3\varepsilon_0}$

A classic



$$\Phi_{in} = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$$

$$\Phi_{out} = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta) - E_0 r \cos\theta$$

B.C. E_0

@ $r=a$ $\Delta\Phi = 0$ $\Delta E_\parallel = 0$ $\Delta D_\perp = \sigma = 0$

$\Phi_{out} = \Phi_{in}$ $$A_l a^l = \frac{B_l}{a^{l+1}} - E_0 a\, \delta_{l,1}$$

$D_{r,out} = D_{r,in}$ $$l A_l a^{l-1} k = -(l+1)\frac{B_l}{a^{l+2}} - E_0\, \delta_{l,1}$$

$l \neq 1$ $$A_l = B_l = 0$$

$l = 1$
$$A_1 - \frac{B_1}{a^3} = -E_0$$
$$kA_1 + \frac{2B_1}{a^3} = -E_0$$

$2\times① + ②$ $\quad (\kappa+2) A_1 = -3E_0 \qquad \boxed{A_1 = \frac{-3E_0}{\kappa+2}}$

$-\kappa\times① + ② \quad (\kappa+2) B_1 = (\kappa-1)E_0 \qquad B_1 = \frac{(\kappa-1)E_0 a^3}{\kappa+2}$

$$\boxed{\Phi_{in} = -\frac{3}{\kappa+2} E_0 r \cos\theta}$$

$$\boxed{\Phi_{out} = -E_0 r\cos\theta + \left(\frac{\kappa-1}{\kappa+2}\right) E_0 a^3 \frac{\cos\theta}{r^2}}$$

$\boxed{\vec{E}_{in} = \frac{3}{\kappa+2}\vec{E}_0}$ $\qquad \kappa\to\infty,\ \vec{E}_{in}\to 0$ conductor $\kappa\to\infty$

$$\vec{P} = \varepsilon_0\chi\vec{E} = \varepsilon_0(\kappa-1)\frac{3}{\kappa+2}\vec{E}_0$$

$$\vec{p} = \frac{4\pi}{3}a^3\cdot\vec{P} = 4\pi\varepsilon_0\cdot\left(\frac{\kappa-1}{\kappa+2}\right)\vec{E}_0 a^3$$

$$\Phi_{out} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad ✓$$

$$\cancel{\int d\Omega\, \hat{r} \left(\frac{1}{r_>} + \frac{\vec{r}\cdot\vec{r}'}{r_>^3} + \frac{3(\hat{r}\cdot\hat{r}')^2 - 1}{2} \frac{r_<^2}{r_>^3} + \cdots \right)}$$

Theorem: $\langle \vec{E}_{in} \rangle = -\frac{1}{3\epsilon_0} \frac{\vec{p}}{V} + \vec{E}_0$

$$\vec{E}_{in} = -\frac{1}{3\cancel{\epsilon_0}} \left(\cancel{\epsilon_0} \chi \vec{E}_{in} \right) + \vec{E}_0$$

$$\vec{E}_{in} + \frac{1}{3}(K-1)\vec{E}_{in} = \left(\frac{K+2}{3}\right)\vec{E}_{in} = \vec{E}_0$$

$$\vec{E}_{in} = \frac{3}{K+2}\vec{E}_0$$