10/26/2005

$$\Phi_{in} = \sum A_\ell r^\ell P_\ell(\cos\theta)$$

$\nabla \cdot \vec{D} = \rho \qquad \nabla \cdot \vec{E} = \rho/\varepsilon$

$$\Phi_{out} = \sum \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta) - E_0 r \cos\theta$$

$(\ell = 1)$.

$$\boxed{\Phi_{in} = - E_0 \left(\frac{3}{\kappa+2}\right) r \cos\theta}$$

$$\Phi_{out} = - E_0 r \cos\theta + \left(\frac{\kappa-1}{\kappa+2}\right) \frac{E_0 a^3}{r^2} \cos\theta$$

$$\vec{P} = \varepsilon_0 \chi \vec{E} = \varepsilon_0 (\kappa - 1) \frac{3}{(\kappa+2)} \vec{E}_0$$

$$\vec{p} = \frac{4\pi}{3} a^3 \vec{P} = 4\pi\varepsilon_0 \left(\frac{\kappa-1}{\kappa+2}\right) \vec{E}_0 a^3$$

$$\Phi_p = \left(\frac{\kappa-1}{\kappa+2}\right) \frac{\vec{p} \cdot \vec{x}}{r^3}.$$

Surface charges

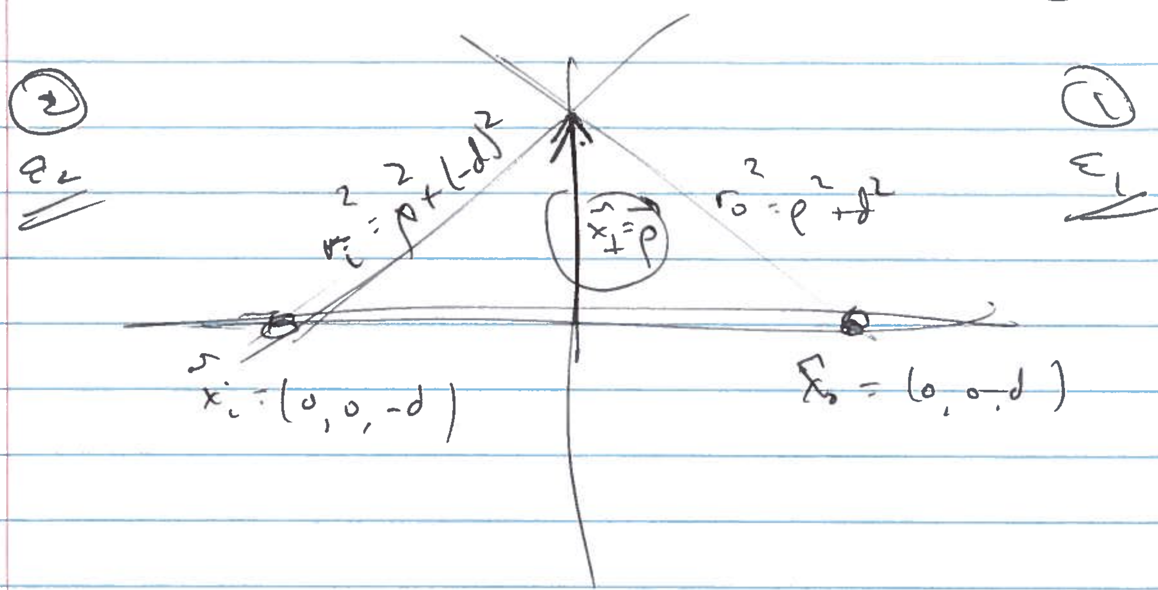
$$\langle \vec{E} \rangle = -\frac{1}{3\varepsilon_0} \frac{\vec{p}}{V} + E_0 = -\frac{1}{3\varepsilon_0} (\varepsilon_0 \chi \vec{E}) + E_0$$

$(\kappa - 1)$

$$\vec{E} \left[1 + \frac{1}{3}(\kappa - 1) \right] = \left[\frac{3 + \kappa - 1}{3} \right] \vec{E}$$

$$= \left(\frac{\kappa+2}{3}\right) \vec{E} = \vec{E}_0$$

$$\sigma = \varepsilon_0 \Delta E_\perp = \varepsilon_0 \left(E_0 - \left(\frac{3}{\kappa+2}\right) E_0 \right) = \varepsilon_0 \left(\frac{\kappa-1}{\kappa+2}\right) 3 E_0 \cos\theta$$



in 1 . $\Phi_1 = \frac{1}{4\pi\varepsilon_1} \frac{q}{|\vec{x} - \vec{x}_0|} + \frac{1}{4\pi\varepsilon_1} \frac{q'}{|\vec{x} - \vec{x}_i'|}$

original $\underline{q}$ plus image $\underline{q'}$ at $\vec{x}_i'$

in 2 $\Phi_2 = \frac{1}{4\pi\varepsilon_0} \frac{q''}{|\vec{x} - \vec{x}_0|}$ @ $\vec{x}_0$, "Filtered" as q''

$\vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot \left(\frac{\vec{E}}{\varepsilon}\right) = \rho/\varepsilon$ (except at boundary.)

only "source" is q_0.

$\vec{E} = -\nabla\Phi$

in 1. $\vec{E} = \frac{1}{4\pi\varepsilon_1} \frac{q(\vec{x} - \vec{x}_0)}{|\vec{x} - \vec{x}_0|^3} + \frac{1}{4\pi\varepsilon_0} \frac{q'(\vec{x} - \vec{x}_i)}{|\vec{x} - \vec{x}_i|^3}$

in 2 $\vec{E} = \frac{1}{4\pi\varepsilon_2} \frac{q''(\vec{x} - \vec{x}_0)}{|\vec{x} - \vec{x}_0|^3}$ $r_0^3 = r_i^3$

at $\underline{z=0}$ $\quad \Delta D_\perp = 0$. $\quad \varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp}$.

$$\varepsilon_1 \left(\frac{1}{4\pi\varepsilon_1} q \frac{(-d)}{r_0^3} + \frac{1}{4\pi\varepsilon_1} q' \frac{(+d)}{r_0^3} \right)$$

$$= \varepsilon_2 \left(\frac{1}{4\pi\varepsilon_2} q'' \frac{(-d)}{r_0^3} \right)$$

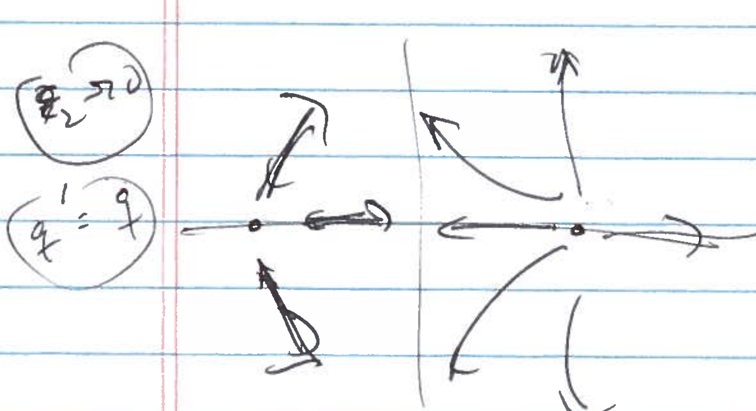
$$q - q' = q''$$

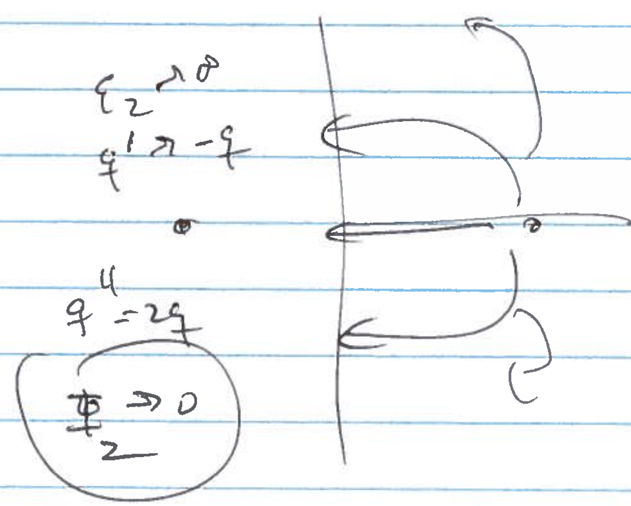
$\Delta E_\parallel = 0$ $\quad \frac{1}{4\pi\varepsilon_1}\left(\frac{q\vec{\rho}}{r_0^3} + \frac{q'\vec{\rho}}{r_0^3} \right) = \frac{1}{4\pi\varepsilon_2} \frac{q''\vec{\rho}}{r_0^3}$

$$\frac{1}{\varepsilon_1}(q + q') = \frac{1}{\varepsilon_2} q''$$

$$q' = -\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1}\right) q \qquad q'' = \frac{2\varepsilon_2}{\varepsilon_2 + \varepsilon_1} q$$

changes sign $\qquad\qquad$ always +.





4

Stored energy $\qquad W = -\int \vec{F}\cdot d\vec{l}$

$W = \int d^3x \; \rho(\vec{x})\,\Phi(\vec{x})$ to insert ρ in given Φ.

$\delta W = \int d^3x \cdot \delta\rho(\vec{x})\,\Phi(\vec{x})$

$= \int d^3x \; \vec{\nabla}\cdot(\delta\vec{D})\,\Phi = \int d^3x \left[\cancel{\vec{\nabla}\cdot(\Phi\,\delta\vec{D})} - \delta\vec{D}\cdot\vec{\nabla}\Phi \right]$

$\boxed{\delta W = \int d^3x \; \vec{E}\cdot\delta\vec{D}}$ w/o $\vec{P}(\vec{E})$, all you can say

linear. $\underline{\vec{D} = \varepsilon\vec{E}}$. $\delta\vec{D}\cdot\vec{E} = \varepsilon\,\delta\vec{E}\cdot\vec{E} = \delta\left(\frac{1}{2}\varepsilon|\vec{E}|^2\right)$

$\boxed{W = \int d^3x \; \frac{1}{2}\varepsilon|\vec{E}|^2}$

$\varepsilon > \varepsilon_0$. $W > W_0$.

Need to work to strain material, as well as move Q's.

5

Fixed Sources (e). — introduce dielectric

$$W_0 = \frac{1}{2}\int d^3x\, \vec{E}_0\cdot\vec{D}_0 \qquad (\vec{D}_0 = \varepsilon_0 \vec{E}_0)$$

$$W_1 = \frac{1}{2}\int d^3x\, \vec{D}\cdot\vec{E} \qquad (\vec{D} = \varepsilon\vec{E})$$

$$\Delta W = W_1 - W_0 = \frac{1}{2}\int d^3x\, (\vec{E}\cdot\vec{D} - \vec{E}_0\cdot\vec{D}_0)$$

$$= \frac{1}{2}\int d^3x \left[(\vec{E}+\vec{E}_0)\cdot(\vec{D}-\vec{D}_0) + \vec{E}\cdot\vec{D}_0 - \vec{D}\cdot\vec{E}_0 \right]$$

$$\hookrightarrow -\vec{\nabla}(\Phi+\Phi_0) \to \vec{\nabla}\cdot(\vec{D}-\vec{D}_0) \to 0.$$

$$\Delta W = \frac{1}{2}\int d^3x \left[\vec{E}\cdot(\varepsilon_0\vec{E}_0) - (\varepsilon\vec{E})\cdot\vec{E}_0 \right]$$

$$= -\frac{1}{2}\int_{\text{object}} d^3x\, (\varepsilon-\varepsilon_0)\vec{E}\cdot\vec{E}_0$$

$$= -\frac{1}{2}\int_{\text{object}} d^3x\, (\vec{D}-\varepsilon_0\vec{E})\cdot\vec{E}_0 = -\frac{1}{2}\int d^3x\, \vec{P}\cdot\vec{E}_0$$

$$\boxed{\Delta W = -\frac{1}{2}\int d^3x\, \vec{P}\cdot\vec{E}_0} \qquad \text{cf. } \underline{W = -\vec{p}\cdot\vec{E}.}$$

small distortions → $\vec{E}_0$ not much changed

→ lowest energy in strongest $\vec{E}_0$.