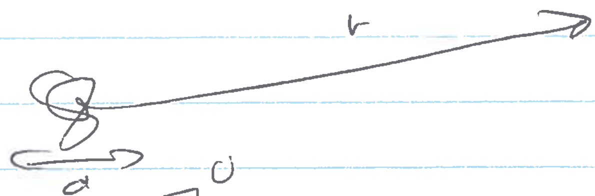
11/9/2015. Localized.



$$\vec{A} = \frac{\mu_0}{4\pi}\int d^3x' \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} \to \left[\cancel{\frac{1}{r}} + \frac{\vec{x}\cdot\vec{x}'}{r^3} + \cdots\right]$$

$$\boxed{\vec{A} \longrightarrow \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{x}}{r^3}} \qquad \boxed{\vec{m} = \int d^3x \, \frac{1}{2}\, \vec{x}\times\vec{J}(\vec{x})} \to I \cdot A \cdot \hat{n}$$

$$\vec{F} = \int d^3x \, \vec{J}\times\vec{B} \qquad (\vec{J} \text{ localized}, \ \vec{B} \text{ smooth})$$

$$= \int d^3x \, \vec{J}(\vec{x}) \times \left[\vec{B}(0) + (\vec{\nabla}_p B_k)_0 \cdot x_p + \cdots\right]$$

$$= \varepsilon_{ijk} (B_k)_0 \cdot \cancel{\int d^3x \, J_j(\vec{x})} + \varepsilon_{ijk} (\nabla_p B_k)_0 \int d^3x \, x_p J_j$$

$$= \varepsilon_{ijk} \left(\frac{\partial B_k}{\partial x_p}\right)_0 \int d^3x \left[\underbrace{\frac{1}{2}(x_p J_j + x_j J_p)}_{\to 0} + \frac{1}{2}(x_p J_j - x_j J_p)\right]$$

$$\varepsilon_{ijk} \int d^3x \, \frac{1}{2}\left[\vec{J}_j (\vec{x}\cdot\vec{\nabla}) - \vec{x}_j (\vec{J}\cdot\vec{\nabla})\right] B_k \Big|_0$$

$$= \varepsilon_{ijk} \underbrace{\int d^3x \, \frac{1}{2}(\vec{x}\times\vec{J})}_{} \cdot \vec{\nabla} B_k \Big|_0$$

$$\boxed{\vec{F} = (\vec{m}\times\vec{\nabla})\times\vec{B}\,\Big|_{\text{center}}}$$

Looks like $U = -\vec{m}\cdot\vec{B}$

$$(\vec{m}\times\vec{\nabla})\times\vec{B} = \vec{\nabla}(\vec{m}\cdot\vec{B}) - \vec{m}(\underbrace{\vec{\nabla}\cdot\vec{B}}_{\to 0}) = -\vec{\nabla}(-\vec{m}\cdot\vec{B})$$

2

$\vec{B}_0$

$$\vec{N} = \sum \vec{r} \times \vec{F} \longrightarrow \int d^3x \ \vec{x} \times (\vec{J} \times \vec{B})$$

$$= \int d^3x \left[\vec{J}(\vec{x} \cdot \vec{B}) - \vec{B}\cancel{(\vec{x} \cdot \vec{J})} \right]$$

$$x_j J_j$$

$$= \int d^3x \ J_i x_j B_j \rightarrow \int d^3x \ \tfrac{1}{2} (J_i x_j - J_j x_i) B_j$$

$$= \int d^3x \ \tfrac{1}{2} (x \times \vec{J}) \times \vec{B} = \vec{m} \times \vec{B}$$

$$\boxed{\vec{N} = \vec{m} \times \vec{B}}$$

compass.

$U = -mB\cos\theta$

$N = mB\sin\theta = -\dfrac{\partial U}{\partial \theta}$ ($\acute{u}$)

$$\vec{x} \times \vec{J} = \vec{x} \times \rho \vec{v} = \rho (\vec{x} \times \vec{v})$$

Swarm of charged particles:

$$\rho = \sum_i q_i \, \delta(\vec{x} - \vec{x}_i(t)).$$

$$\vec{J}(x) = \sum_i q \vec{v}_i \, \delta(\vec{x} - \vec{x}_i(t)) \qquad \vec{v}_i = \frac{d\vec{x}_i}{dt}.$$

$$\vec{m} = \int d^3x \, \frac{1}{2} \vec{x} \times \vec{J} \rightarrow \frac{1}{2} \sum_i q_i \vec{x}_i \times \vec{v}_i$$

$$= \frac{1}{2} \sum_i \left(\frac{q_i}{m_i}\right) \vec{x}_i \times m_i \vec{v}_i$$

$$\vec{m} = \frac{q}{2m} \vec{L} \qquad \frac{q_i}{m_i} = \frac{q}{m} \qquad \text{orbital}$$

Spin $\rightarrow \vec{\mu} = g \cdot \frac{q}{2m} \vec{S}$

~~rela~~ relativistically $g = 2$ "Thomas precession"

QED. $g = 2\left(1 + \frac{\alpha}{\pi} + \cdots\right)$

$$= 2(1.00115965218076$$
$$\pm \quad 00000027 \qquad (\pm 0.27 \times 10^{-12}$$

QCD $\quad 1.001 \rightarrow \quad 2.79284\,7$

$$- 1.913042$$

Volume average $\vec{B}(\vec{x})$.

$$\langle \vec{B}(\vec{x}) \rangle_R = \frac{1}{\frac{4\pi}{3}R^3} \int_{r<R} d^3x \, \vec{\nabla} \times \vec{A} = \frac{1}{V} \oint_{r=R} R^2 d\Omega \, \hat{r} \times \vec{A}$$

$$= \frac{1}{V} \int R^2 d\Omega \cdot \hat{r} \times \frac{\mu_0}{4\pi} \int d^3x' \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|}$$

$$= -\frac{1}{V} \frac{\mu_0}{4\pi} R^2 \int d^3x' \, \vec{J}(\vec{x}') \times \int d\Omega \frac{\hat{r}}{|\vec{x}-\vec{x}'|} \rightarrow \frac{4\pi}{3} \frac{r_<}{r_>^2} \hat{r}'$$

$$= \frac{1}{\frac{4\pi}{3}R^3} \cdot \frac{\mu_0}{4\pi} \left[\frac{4\pi}{3} \cdot 2 \int_0^R d^3x' \, \frac{1}{2} \vec{x}' \times \vec{J}(\vec{x}') \right.$$

$$\left. + \frac{4\pi}{3} \cdot R^3 \cdot \int_R^\infty d^3x' \, \vec{J}(\vec{x}') \times \left(\frac{0-\vec{x}'}{|0-\vec{x}'|^3} \right) \right.$$

$$\boxed{\langle \vec{B} \rangle_R = \frac{2\mu_0}{3} \frac{\vec{m}_{in}}{V} + \vec{B}_{out}(0)}$$

cf. $-\frac{\epsilon_0}{3} \vec{p}_{in}/V$.

$$\boxed{\vec{B}_{dipole} = \frac{\mu_0}{4\pi} \cdot \frac{3\hat{r}(\hat{r}\cdot\vec{m}) - \vec{m}}{r^3} + \frac{2\mu_0}{3} \vec{m} \, \delta^3(\vec{x})}$$

5

$$U = -\vec{m}_e \cdot \vec{B}_p = -\frac{\mu_0}{4\pi} \frac{3(\hat{r}\cdot\vec{m}_e)(\hat{r}\cdot\vec{m}_p) - \vec{m}_e\cdot\vec{m}_p}{r^3}$$

$$- \frac{2\mu_0}{3} \vec{m}_e \cdot \vec{m}_p \, \delta^{(3)}(\vec{x})$$

$$\langle\psi|U|\psi\rangle = \int d^3x \, |\psi|^2 U = -\frac{2\mu_0}{3} \vec{m}_e \cdot \vec{m}_p \cdot |\psi(0)|^2$$

$$\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \qquad |\psi(0)|^2 = \frac{1}{\pi a_0^3}$$

$$\vec{\mu}_p \cdot \vec{\mu}_e = \cancel{2} g_e \cdot \frac{e}{\cancel{2} m_e} \cdot \cancel{2} g_p \cdot \frac{e}{\cancel{2} m_p} \; \hbar^2 \frac{1}{2}\left(\underline{\underline{s(s+1)}} - \frac{3}{4} - \frac{3}{4}\right)$$

$$\Delta E = \frac{2\mu_0}{3} \cdot \left(\frac{\varepsilon_0}{\varepsilon_0}\right) \frac{1}{\pi} \left(\frac{e^2}{4\pi\varepsilon_0} \frac{m_e}{\hbar^2}\right)^3 g_p g_e \frac{e^2 \hbar^2}{m_p m_e} \cdot 2 \cdot 2$$

$$= \frac{8}{3} \left(\frac{e^2/4\pi\varepsilon_0}{\hbar c}\right)^4 g_p g_e \left(\frac{m_e}{m_p}\right) m_e$$

$$\frac{8}{3} \cdot \left(\frac{1}{137.036}\right)^4 \binom{1.00115}{2.79 \cdot} \left(\frac{0.511}{1836.15}\right) (0.511 \text{ MeV})$$

$\rightarrow$ 21.07 cm 1420 MHz

Materials $\vec{J}$: volume current density $\vec{m}(\vec{x})$ = dipole moment density

magnetization

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int d^3x' \left[\frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} + \frac{\vec{m}(\vec{x}') \times (\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3} \right] + \cancel{\cdots}$$

① First, most important correction

② "Theorem" $\langle \vec{r} \rangle_{12}$

③ Quadrupole ordering difficult (to produce, to compute)

$$\int d^3x' \, \vec{m}(\vec{x}') \times \left(\frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|} \right) = \int d^3x' \, \vec{M} \times \vec{\nabla}' \left(\frac{1}{|\vec{x}-\vec{x}'|} \right)$$

$$= \int d^3x' \, \varepsilon_{ijk} \left[\underbrace{\partial_k' \left(\frac{m_j}{|\vec{x}-\vec{x}'|} \right)}_{\text{Surface} \to 0} - (\vec{\nabla}_k' m_j) \left(\frac{1}{|\vec{x}-\vec{x}'|} \right) \right]$$

$$= - \int d^3x' \frac{\varepsilon_{ijk} \partial_k' m_j}{|\vec{x}-\vec{x}'|} = + \int d^3x' \frac{(\vec{\nabla}' \times \vec{m})}{|\vec{x}-\vec{x}'|}$$

$$\boxed{\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int d^3x' \frac{[\vec{J}(\vec{x}') + \vec{\nabla}' \times \vec{m}]}{|\vec{x}-\vec{x}'|}}$$

Source = $\underline{\vec{J} + \vec{\nabla} \times \vec{m}}$ $\quad \underline{\vec{J}_m = \vec{\nabla} \times \vec{m}}$ (cf. $\rho_p = -\vec{\nabla} \cdot \vec{P}$)