11/13/15

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int d^3x' \left[\frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} + \vec{m}(\vec{x}') \times \frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3} \right]$$

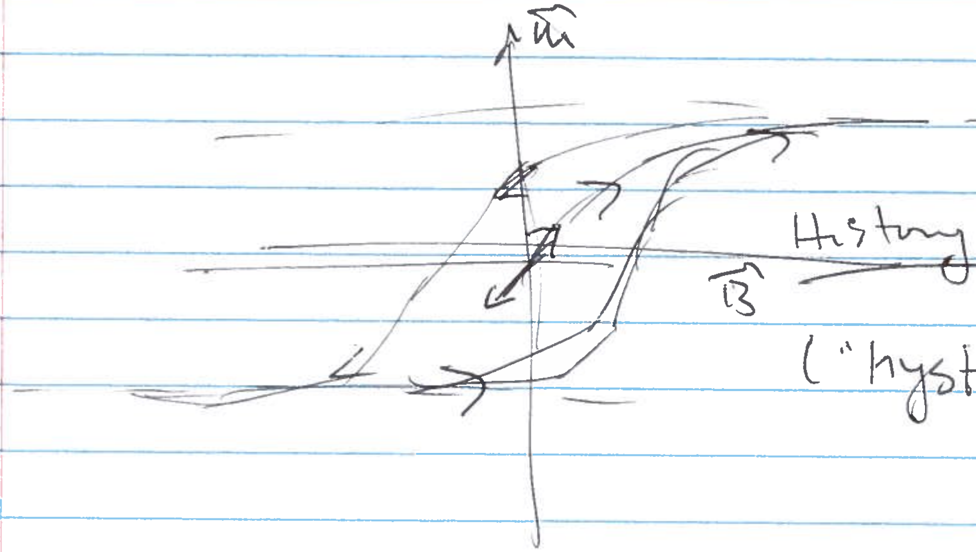
(parts) $\rightarrow \dfrac{\mu_0}{4\pi}\displaystyle\int d^3x' \frac{(\vec{J} + \vec{\nabla}'\times\vec{m})}{|\vec{x}-\vec{x}'|}$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0(\vec{J} + \vec{\nabla}\times\vec{m})} \qquad \vec{J}_m = \vec{\nabla}\times\vec{m}$$

$$\vec{\nabla}\times\left(\frac{1}{\mu_0}\vec{B} - \vec{m}\right) = \vec{J} \qquad \boxed{\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}}$$

$$\vec{\nabla}\times\vec{H} = \vec{J}$$
$$\vec{\nabla}\cdot\vec{B} = 0 \qquad (\vec{B} = \vec{\nabla}\times\vec{A})$$

Still need $\vec{m} = \vec{m}(\vec{B})$ or some such.



(saturates)

History-dependent

("hysteresis")

Idealizations

① "Hard" magnetization $\vec{m} = \vec{m}_0$
independent of (w/o) applied $\vec{B}$.

② local, linear, isotropic $\vec{B} = \mu\vec{H} = \mu_0 \kappa \vec{H}$

$$\vec{M} = \frac{1}{\mu_0}\vec{B} - \vec{H} = \left(\frac{\mu}{\mu_0} - 1\right)\vec{H} = (\kappa - 1)\vec{H} = \chi_m \vec{H}$$

$\kappa > 1$. $\chi > 0$ "paramagnetic" (stronger)

$\kappa < 1$ $\chi < 0$ "diamagnetic" (weaker)

χ_e often of order 1 - several. (unpaired) + (paired, orbital)

χ_m: aligning weak atomic dipoles. $\pm 10^{-4}, 10^{-5}$

③ Ferromagnetic, weak field linear $\kappa \sim 10^{4} - 10^{6}$

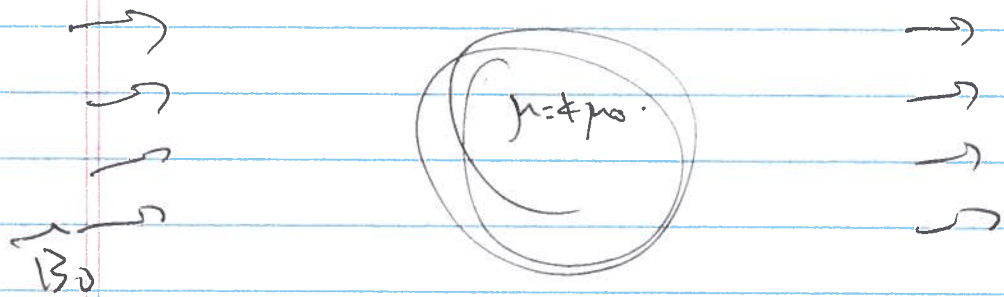
μ-metal 77% Ni 16% Fe 5% Cu. 2% Cr
mb.

[① + ②]

Boundary conditions.

$\vec{\nabla}\cdot\vec{B} = 0$ $\qquad$ $\boxed{\Delta B_\perp = 0}$

$\vec{\nabla}\times\vec{H} = \vec{J}$ $\qquad$ $\boxed{\hat{n} \times \Delta\vec{H}_\parallel = \vec{K}}$



If we knew $\vec{B}_{in}$ = constant

$$\langle \vec{B}_{in} \rangle = \frac{2\mu_0}{3}\vec{M}_{in} + \vec{B}_0$$

$$= \frac{2\mu_0}{3}\chi\vec{H}_{in} + \vec{B}_0 = \frac{2\cancel{\mu_0}}{3}(k-1)\frac{\vec{B}_{in}}{k\cancel{\mu_0}}$$

$$\vec{B}_{in}\left[1 - \frac{2}{3}\frac{(k-1)}{k}\right] = \vec{B}_{in}\left(\frac{3k-2k+2}{3k}\right) = \vec{B}_0$$

$$\boxed{\vec{B}_{in} = \frac{3k}{k+2}\vec{B}_0}$$

"conductor" $k \to 0$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{B} = \vec{\nabla}\times(\vec{H}) = 0$ (except at $r=a$ $\rightarrow$ B.C)

$\vec{\nabla}\cdot\vec{B} = \vec{\nabla}\cdot(\mu\vec{H}) = \mu\,\vec{\nabla}\cdot\vec{H} = 0$ except at $r=a$.

$\vec{\nabla}\times\vec{H} = \vec{J} = 0$ $\quad \boxed{\vec{H} = -\vec{\nabla}\Phi}$

$\nabla^2\Phi = 0$

$$\Phi_{in} = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$$

$$\Phi_{out} = -\frac{1}{\mu_0}\underline{B_0 r\cos\theta} + \sum_{l=0}^{\infty}\frac{B_l}{r^{l+1}}P_l(\cos\theta)$$

$\hookrightarrow$ $l=1$ $\rightarrow$ B_{in} = constant

$\Delta H_{\parallel} = 0$, $\Delta\Phi = 0$ $\qquad A_l a^l = -\frac{1}{\mu_0}B_0 a\,\delta_{l,1} + \frac{B_l}{a^{l+1}}$

$\Delta B_{\perp} = 0$ $\qquad \mu\left.\frac{\partial\Phi_{in}}{\partial r}\right|_{r=a} = \mu_0\left.\frac{\partial\Phi_{out}}{\partial r}\right|_{r=a}$

$$K\cdot l\cdot A_l a^{l-1} = -\frac{1}{\mu_0}B_0\,\delta_{l,1} - (l+1)\frac{B_l}{a^{l+2}}$$

$l\neq 1$ $\quad A_l = B_l = 0$

$(l=1)$ $$A_1 = -\frac{1}{\mu_0} B_0 + \frac{B_1}{a^3} \qquad (1)$$

$$KA_1 = -\frac{1}{\mu_0} B_0 - \frac{2B_1}{a^3} \qquad (2)$$

$2(1)+(2)$ $$(K+2)A_2 = -\frac{3}{\mu_0} B_0$$

$$\boxed{A_1 = -\left(\frac{3}{K+2}\right)\frac{B_0}{\mu_0}} \qquad \vec{H}_{in} = \frac{3}{K+2}\hat{H}_0 \qquad \hat{B}_{in} = \frac{3k}{K+2}\vec{B}_0$$

$(2) - K(1)$ $$(2+K)\frac{B_1}{a^3} = -\frac{1}{\mu_0} B_0 (1-K)$$

$$\boxed{B_1 = \left(\frac{K-1}{K+2}\right)\frac{B_0 a^3}{\mu_0}}$$

$$\vec{m} = \left(\frac{4\pi}{3}a^3\right)(K-1)\frac{3\vec{B}_0}{(K+2)\mu_0}$$

Cylinder

I. $\rho < a$ $\quad \Phi = A_1 \rho \cos\phi$

II. $a < \rho < b$ $\quad \Phi = B_1 \rho \cos\phi + \frac{C_1}{\rho} \cos\phi$

III. $\rho > b$ $\quad \Phi = -\frac{B_0}{(\mu_0)} \rho \cos\phi + \frac{D_1}{\rho} \cos\phi$

$\Delta B_\perp = 0 \quad \Delta H_\parallel = 0 \quad (\rho = a), (\rho = b).$

$$\rightarrow \boxed{B_{in} = B_0 \cdot \frac{4K}{(K+1)^2 - \left(\frac{a^2}{b^2}\right)(K-1)^2}}$$

$K >> 1. \rightarrow \cancel{\frac{4}{K\left(1 - \frac{a^2}{b^2}\right)}}$

conductor $K \to 0$.