11/16/15. linear μ = much like linear ε.

"Hard" $\vec{m}_0$. cylindrical bar magnet

Two ways: I. Surface Current

$$\vec{\nabla}\times\vec{B} = \mu_0\left(\cancel{\vec{J}} + \vec{\nabla}\times\vec{m}\right) = \left(-\hat{n}\,\delta(\text{surface})\right)\times\vec{M}_0$$

ends. $\hat{n} = \pm\hat{z}$ $\quad \hat{n}\times m_0 = 0$.

sides $\quad \hat{n} = \hat{\rho} \quad (-\hat{\rho})\times(m_0\hat{z}) = m_0\hat{\phi}$

§5.3. Biot savart w/ $\vec{K}$

$$\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi}\int \vec{K}(\vec{x}')\,d^2a' \times \frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}$$

$$= \frac{\mu_0}{4\pi}\int \frac{m_0\hat{\phi}'\cdot b\,d\phi'\,dz' \times \left(\cancel{\rho\hat{\rho}} + z\hat{z} - a\hat{\rho}' - z'\hat{z}\right)}{\left(\cancel{\rho^2} + a^2 - \cancel{2\rho a\cos(\phi-\phi')} + (z-z')^2\right)^{3/2}}$$

$\rho = 0$

$$\vec{B}(z) = \frac{\mu_0}{4\pi} M_0 a \int_{-L/2}^{L/2} dz' \int d\phi' \frac{\cancel{(z-z')\hat{\rho}'} + a\hat{z}}{\left(\rho^2 + (z-z')^2\right)^{3/2}}$$

$$= \frac{\mu_0}{4\pi}\cdot M_0\, 2\pi a^2 \int_{-L/2}^{L/2} \frac{dz'}{\left(a^2+(z-z')^2\right)^{3/2}}$$

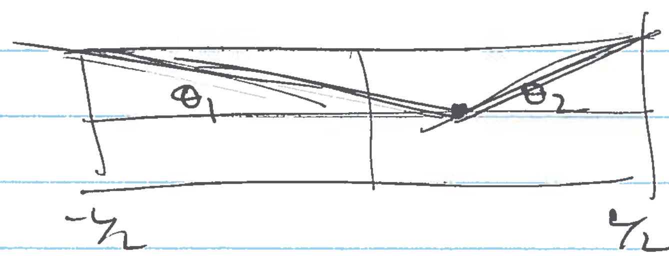
(2)

Let: $z' - z = a \tan\psi$. $\quad \tan\psi_1 = \dfrac{a}{-\frac{L}{2} - z} \quad \tan\psi_2 = \dfrac{a}{\frac{L}{2} - z}$

$$\vec{B} = \frac{1}{2}\mu_0 M_0 \hat{z} \cdot \cancel{a^2} \int_{\psi_1}^{\psi_2} \frac{a \cdot \sec^2\psi \, d\psi}{(\cancel{a^2} + \cancel{a^2}\tan^2\psi)^{3/2}}$$

$$= \frac{1}{2}\mu_0 \vec{M_0} \left(\sin\psi_2 - \sin\psi_1\right)$$

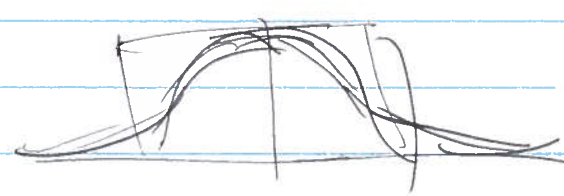
$$\vec{B} = \frac{1}{2}\mu_0 \vec{M_0} \left(\frac{\frac{L}{2} + z}{\sqrt{a^2 + \left(\frac{L}{2} + z\right)^2}} + \frac{\frac{L}{2} - z}{\sqrt{a^2 + \left(\frac{L}{2} + z\right)^2}} \right)$$



$$\vec{B} = \frac{1}{2}\mu_0 \vec{M_0} \left(\cos\theta_1 + \cos\theta_2\right)$$

Long (not near ends). $\theta_1, \theta_2 \to 0$ $\quad \boxed{\vec{B} \to \mu_0 \vec{M_0}}$

center: $\dfrac{2\,(L/2)}{\sqrt{a^2 + (L/2)^2}}$



far outside: nearly cancel. $+\dfrac{z}{|z|} - \dfrac{z}{|z|}$

III. Surface charge   Electrostatic analogy:

No $\vec{J}$ $\vec{\nabla}\times\vec{H}=0$ $\vec{\nabla}\cdot\vec{H}=\vec{\nabla}\cdot\left(\frac{1}{\mu_0}\vec{B}-\vec{m}\right)=-\vec{\nabla}\cdot\vec{M}$

$-\vec{\nabla}\cdot\vec{M}=+\hat{n}\cdot\vec{M}_0\cdot\delta(\text{surface})$.

"σ_m" $=+M_0\ (z=+L/2)$ "σ_m" $=-M_0\ (z=-L/2)$.

Disk of charge (at $z=0$), on-axis.

$$\Phi_m=\frac{1}{4\pi}\int d^2a'\,\frac{\sigma}{|\vec{x}-\vec{x}'|}=\frac{M_0}{4\pi}\int_0^a\frac{2\pi\rho'\,d\rho'}{\sqrt{\rho'^2+z^2}}$$

$$=\frac{1}{2}M_0\cdot\left(\sqrt{a^2+z^2}-\sqrt{z^2}\right)\quad(\text{done before})$$

$$\vec{H}=-\vec{\nabla}\Phi_m=\frac{1}{2}M_0\hat{z}\left(\frac{z}{\sqrt{z^2}}-\frac{z}{\sqrt{a^2+z^2}}\right)$$

$(\hat{n})$.

$\pm L/2$

$$\vec{H}=\frac{1}{2}\vec{M}_0\left[\left(\frac{z-L/2}{|z-L/2|}-\frac{(z-L/2)}{\sqrt{a^2+(L/2-z)^2}}\right)-\left(\frac{z+L/2}{|z+L/2|}-\frac{(z+L/2)}{\sqrt{a^2+(L/2+z)^2}}\right)\right]$$

± 1

$\underline{z < -L/2}$ $\quad \dfrac{z-L/2}{|z-L/2|} = -1 \quad \dfrac{z+L/2}{|z+L/2|} = -1$

$z > L/2. \quad \dfrac{z-L/2}{|z-L/2|} = +1 \quad \dfrac{z+L/2}{|z+L/2|} = +1$

cancel

$\underline{-L/2 < z < L/2}$ $\quad \dfrac{z-L/2}{|z-L/2|} = -1 \quad \dfrac{z+L/2}{|z+L/2|} = +1$

$$\rightarrow \tfrac{1}{2}\vec{M}_0\left((-1) - (1)\right) = -\vec{M}_0 .$$

$\cancel{\vec{B} = \frac{1}{\mu_0}}$ $\quad \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}_0$

$\boxed{\vec{B} = \mu_0(\vec{H} + \vec{M}_0)}$

$\underline{\text{outside}}$, $\vec{B} = \mu_0 \vec{H}$

$\underline{\text{inside}}$. $\vec{B} = \mu_0(\vec{H} + \vec{M}_0)$

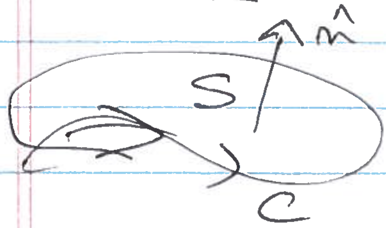
$$\boxed{\vec{B} = \tfrac{1}{2}\mu_0 \vec{M}_0 \left(\frac{z+L/2}{\sqrt{a^2+(L/2+z)^2}} - \frac{(z-L/2)}{\sqrt{a^2+(z-L/2)^2}} \right)}$$

same

far away. $\vec{B} \rightarrow \tfrac{1}{2}\mu_0 \vec{M}_0 \left(\frac{1}{2} \cdot \frac{a^2 L}{z^3} \right) = \frac{\mu_0}{4\pi} \cdot \frac{\pi b^2 L \cdot \vec{M}_0}{z^3}$

$\underline{\text{dipole moment}}$.

Energy ↔ changing $\vec{B}$ Induction



$$\Phi = \int_S d^2a\, \hat{n}\cdot\vec{B}$$

$$\mathcal{E} = \oint_C \vec{E}\cdot\vec{dl}$$

open surface right-hand rule

Faraday's law.

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$

Stationary: $\frac{d\Phi}{dt} = \frac{d}{dt}\int d^2a\, \hat{n}\cdot\vec{B} = \int d^2a\, \hat{n}\cdot\left(\frac{\partial \vec{B}}{\partial t}\right)$

$$\mathcal{E} = \oint \vec{E}\cdot\vec{dl} = \int d^2a\, \hat{n}\cdot(\vec{\nabla}\times\vec{E}).$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

moving: $\frac{d\vec{B}}{dt} = \frac{\partial\vec{B}}{\partial t} + \frac{\partial\vec{B}}{\partial x_i}\frac{dx_i}{dt} = \frac{\partial\vec{B}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{B}$

$$\vec{\nabla}\times(\vec{v}\times\vec{B}) = \vec{v}\,\cancel{(\vec{\nabla}\cdot\vec{B})} - (\vec{v}\cdot\vec{\nabla})\vec{B}$$

$$\frac{d\Phi}{dt} = \frac{d}{dt}\int_S d^2a\, \hat{n}\cdot\vec{B} = \int_S d^2a\, \hat{n}\cdot\left(\frac{\partial\vec{B}}{\partial t} - \vec{\nabla}\times(\vec{v}\times\vec{B})\right)$$

$$= \int d^2a\, \hat{n}\cdot(\vec{\nabla}\times\vec{E})$$

$$\int d^2a\ \hat{n}\cdot\frac{\partial\vec{B}}{\partial t} = -\int d^2a\ \hat{n}\cdot\left(\vec{\nabla}\times\left(\vec{E}' + \vec{v}\times\vec{B}\right)\right)$$

comoving with loop $\boxed{\vec{E}'_{\text{comoving}} = \vec{E}_{\text{lab}} + \vec{v}\times\vec{B}}$

$\vec{B}' = \vec{B}$ ok. for $v \ll c$, will revisit in spring.

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$ Faraday's law.

$$\boxed{\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) = q\vec{E}'}$$

Add to : $\vec{\nabla}\cdot\vec{B} = 0 \longrightarrow \boxed{\vec{B} = \vec{\nabla}\times\vec{A}}$

No longer $\vec{\nabla}\times\vec{E} = 0$ $\quad \vec{E} \neq -\vec{\nabla}\phi$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{A}) = -\vec{\nabla}\times\left(\frac{\partial\vec{A}}{\partial t}\right)$$

$$\vec{\nabla}\times\left(\vec{E} + \frac{\partial\vec{A}}{\partial t}\right) = 0 \qquad \vec{E} + \frac{\partial\vec{A}}{\partial t} = -\vec{\nabla}\Phi$$

$$\boxed{\vec{E} = -\vec{\nabla}\Phi - \frac{\partial\vec{A}}{\partial t}}$$