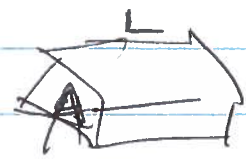
11/18/2015 emf → current in resistive circuit.

Conductor : Ohm's law. $\vec{J} = \sigma \vec{E}$



$V = E \cdot L$

$I = J \cdot A$

$V = \left(\frac{J}{\sigma}\right) \cdot L = \left(\frac{I}{A}\right)\frac{L}{\sigma} = I \cdot \left(\frac{L}{\sigma A}\right) = IR$

$V = IR$ $\quad R = \frac{1}{\sigma}\frac{L}{A} = \rho \frac{L}{A}$ $\quad \rho = \frac{1}{\sigma}$

σ : conductivity $\quad \rho$: resistivity $= \sigma^{-1}$

$\sigma_{Cu} = 5.81 \times 10^{7}\ (\Omega \cdot m)^{-1}$ $\quad$ Siemen / meter

$\sigma_{Ag} = 6.30 \times 10^{7}\ (\Omega \cdot m)^{-1}$

Au. 4.1×10^{7}

Al 3.5×10^{7}

graphene 10^{8} (in sheet).

teflon 10^{-25} $\quad 10^{-24 \pm 1}$

glass $10^{-13 \pm 2}$

dry wood. $10^{-15 \pm 1}$

damp wood. $10^{-3} - 10^{-4}$.

sea water 4.80

~~distilled~~ deionized. 5.5×10^{-6}

$\vec{B}$ and conductors.

$\vec{\nabla}\cdot\vec{E} = 0$: (no charge) $\quad \vec{E} = -\frac{\partial \vec{A}}{\partial t} \quad \vec{\nabla}\cdot\vec{A} = 0$

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$$
$$= \mu\vec{J} = \mu\sigma\vec{E} = -\mu\sigma\frac{\partial\vec{A}}{\partial t}.$$

$$\boxed{\nabla^2\vec{A} = \mu\sigma\cdot\frac{\partial\vec{A}}{\partial t}}$$

$\frac{\partial}{\partial t} \Rightarrow \boxed{\nabla^2\vec{E} = \mu\sigma\cdot\frac{\partial\vec{E}}{\partial t}}$ $\qquad \vec{\nabla}\times\vec{B}$ or $\sigma\vec{E} \rightarrow$ $\boxed{\nabla^2\vec{J} = \mu\sigma\frac{\partial\vec{J}}{\partial t}}$

$\vec{\nabla}\times \rightarrow \boxed{\nabla^2\vec{B} = \mu\sigma\frac{\partial\vec{B}}{\partial t}}$

(diffusion). $\nabla^2\psi = \mu\sigma\cdot\frac{\partial\psi}{\partial t}$. "parabolic"

prototype solution $\boxed{\psi(\vec{x},t) = \frac{1}{(2\pi t/\tau)^{d/2}}\, e^{-\frac{1}{2}\frac{x^2/L^2}{t/\tau}}}$

$(t\to 0)$ $\delta(\vec{x})$ $\qquad t>0$. spreads, $\Delta x \sim \sqrt{t}$.

$$\frac{\nabla^2\psi}{\psi} = \frac{\tau^2 x^2}{L^4 t^2} - \frac{d\tau}{L^2 t}$$

$$\frac{\frac{\partial\psi}{\partial t}}{\psi} = \frac{\tau x^2}{2L^2 t^2} - \frac{d}{2t}$$

$$\left.\right\} \quad \frac{\nabla^2\psi}{\frac{\partial\psi}{\partial t}} = \boxed{\mu\sigma = \frac{2\tau}{L^2}}$$

③

$$\tau = \frac{1}{2}\mu\sigma \cdot L^2$$

Diffusion .. steak twice as thick → (time) ∝ (length)^2

×4

twice as heavy $L \sim 2^{1/3} = 1.25992$

$t \sim 2^{2/3} = 1.58740.$

$t \ll \tau$. Fields "frozen"

$\tau \gg t$. fields diffuse away.

$\mu \sim \mu_0 = 4\pi \times 10^{-7}$

$\sigma = \sigma_{Cu} = 5.81 \times 10^7 \, (\Omega m)^{-1}$

$L = 1\,cm = 10^{-2}\,m$

$\frac{1}{2} \cdot 4\pi \cdot 10^{-7} \cdot 5.81 \cdot 10^{7} \cdot (10^{-2})^2$

$\simeq 12\pi \cdot 10^{-4} = 3.6\,ms$

Earth core:

$\sigma \sim 10^7$

$L \sim \frac{1}{10} R_\oplus : 640\,km$

$\frac{1}{2} \cdot 4\pi \cdot 10^{-7} \cdot 10^{7} \cdot (640 \times 10^3)^2$

$= 2\pi \cdot 40 \cdot 10^{10}$

$= 8 \cdot 10^4 \cdot (\pi \cdot 10^7) = 80{,}000\,y$

Earth's field needs dynamo

Sun. $\gg 10^{10}\,y$ $2\pi \cdot (700{,}000\,km)^2 = 6 \cdot 50 \cdot 10^{16} = 3 \cdot 10^{18} = 10^{11}\,y.$

($R_\odot \sim 700{,}000\,km$)

Sea water (1 m ohms) (wikipedia).

$\frac{1}{2}\mu_0 \sigma L^2 = (6)(10^{-7})(5) = \mu s$??

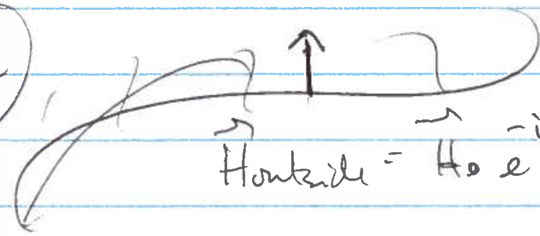
large conductivity: $\vec{E}_{inside}=0$ $\vec{B}_{inside}$ doesn't change

$\vec\nabla\cdot\vec B=0$ $\Delta B_\perp=0$ → $B_\perp|_S=0$

$\vec\nabla\times\vec H=\vec J$. $\Delta H_\parallel = K$.

finite conductivity (K) leaks into interior "skin depth"

wire



$\vec H_{outside}=\vec H_0 e^{-i\omega t}$.   (pic)

inside $\vec H = \vec H(\xi) e^{-i\omega t}$.

$$\nabla^2\vec H = \frac{\partial^2 \vec H}{\partial\xi^2} + \text{(~~other directions "slowly varying"~~)}$$

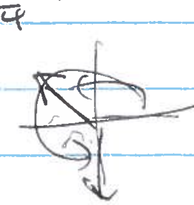
$$\left(\frac{\partial^2}{\partial y^2}\right) \quad \left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)$$

$$= \mu\sigma\frac{\partial \vec H}{\partial t} = \mu\sigma(-i\omega)\vec H$$

$$\hat H(\xi) = \hat H_0 e^{\alpha\xi} \qquad \alpha^2 = -i\mu\sigma\omega$$

$$\alpha = \pm\left(\frac{-1+i}{\sqrt2}\right)\sqrt{\mu\sigma\omega} \qquad (-i) = e^{\frac{3\pi}{2}i + 2\pi i\cdot n}$$

$$(-i)^{1/2} = e^{\frac{3\pi i}{4}+in\pi}$$

$= \pm$ 

physics: decays into material

→ (+).

$$\alpha = (-1+i)\sqrt{\frac{\mu\sigma\omega}{2}} = \frac{-1+i}{\delta} \qquad \delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$

$$\vec{H} = \vec{H}_0 \, e^{-i\xi/\delta} \, e^{-\xi/\delta} \, e^{-i\omega t}$$

$$\vec{H} = \vec{H}_0 \, e^{-\xi/\delta} \cos\left(\xi/\delta - \omega t\right)$$

$\Delta H_\perp = 0$ $\quad \vec{\nabla}\cdot\vec{H} = 0$ inside outside $\rightarrow \vec{\nabla}\cdot\vec{H} = (\alpha)\,\hat{n}\cdot\vec{H}_0 \cdots = 0 \quad \vec{H}_0 \parallel$ surface

$$\vec{\nabla}\times\vec{H} = \left(\hat{n}\frac{\partial}{\partial\xi}\right)\times\left(\vec{H}_0 \, e^{\alpha\xi} \, e^{-i\omega t}\right)$$

$$\vec{J} = \hat{n}\times\vec{H}_0 \, \alpha \, e^{\alpha\xi} \, e^{-i\omega t} = \frac{(-1+i)}{\delta}(\hat{n}\times\vec{H}_0) \, e^{\alpha\xi} \, e^{-i\omega t}$$

"Surface" current layer $\vec{J} \perp \hat{n} \quad \vec{J} \perp \vec{H}_0$

distributed over thickness $\sim \delta$.

$\frac{\pi}{4}$ out of phase

$$\int_0^\infty d\xi \, \vec{J} = (\hat{n}\times\vec{H}_0) = \vec{K}.$$

$f = 10\ GHz$. (microwave).

$$\lambda = \frac{c}{f} = \frac{3\times10^8\ m/s}{10^{10}\ s^{-1}} = 3cm.$$

$\omega = 6.3\times10^{10}\ rad\ s^{-1} = 2\pi\cdot10^{10}$

$\mu = \mu_0 = 4\pi\times10^{-7}$

$\sigma = 5.8\times10^7$

$$\sqrt{\frac{2}{2\pi\cdot10^{10}\cdot4\pi\cdot10^{-7}\cdot5.8\cdot10^7}}$$

$$= \frac{1}{2\pi}\frac{1}{\sqrt{5.8}}10^{-5}\ m = 0.66\ \mu m$$

Seawater . $\delta = \dfrac{240\ m}{\sqrt{f(Hz)}}$ (230 m)

ELF Submarine communication 30-300 Hz

$$\frac{3\times10^8\ m/s}{30\ Hz} = 10^7\ m = 10^4\ km$$

Seafarer (UP = 53 km.)

$$75\ Hz \rightarrow \frac{3\times10^8\ m/s}{75\ Hz} = 4\times10^6\ m = 4000\ km.$$

$$\frac{240\ m}{\sqrt{75}} = 30\ m = 100\ ft$$

Russian ZEVS . (60 km) (82 Hz) measurement

michigan . (52 km) (76 Hz)