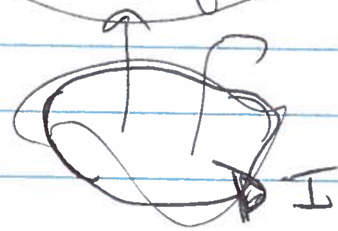
11/20/15

Work . $\underline{\vec{F}_B = q\vec{v}\times\vec{B}}$  $\vec{v}\cdot\vec{F} = 0$

"magnetic fields do no work"

Changing $\vec{B}$ ~~does~~ needs work! Current flow against $\mathcal{E}$)



$$\mathcal{E} = -\frac{d\Phi}{dt}.$$

$$\frac{dW}{dt} = -I\mathcal{E} \qquad \left(\frac{W}{q}\right)\left(\frac{q}{t}\right).$$

$$\delta W = +I\,\delta\Phi = I\cdot\int_S d^2a\,\hat{n}\cdot\delta\vec{B}$$

$$= I\int_S d^2a\,\hat{n}\cdot(\vec{\nabla}\times\delta\vec{A}) = \oint_C I\,d\vec{l}\cdot\delta\vec{A}$$

"many loops" $\rightarrow$ $\boxed{\delta W = \int d^3x\,\vec{J}\cdot\delta\vec{A}}$ (5.144)

[cf. $\delta W = \int d^3x\,\delta\rho\cdot\Phi$ (4.84)]

$\vec{J} = \vec{\nabla}\times\vec{H}$

$$\underline{\vec{\nabla}\cdot(\vec{H}\times\delta\vec{A})} = \varepsilon_{ijk}\nabla_i H_j\,\delta A_k$$

$$= \varepsilon_{ijk}\left((\nabla_i H_j)\,\delta A_k + H_j(\nabla_i\,\delta A_k)\right)$$

$$= \delta\vec{A}\cdot(\underbrace{\vec{\nabla}\times\vec{H}}_{\vec{J}}) - \vec{H}\cdot(\underbrace{\vec{\nabla}\times\delta\vec{A}}_{\delta\vec{B}}).$$

$$\int_V d^3x\, \vec{J}\cdot\delta\vec{A} = \int d^3x \left\{ \vec{\nabla}\cdot(\vec{H}\times\delta\vec{A}) + \underline{\vec{H}\cdot\delta\vec{B}} \right\}$$

Surface $\to 0$

$$\boxed{\delta W = \int d^3x\, \vec{H}\cdot\delta\vec{B}} \quad (5.147)$$

$$\left( \text{cf. } \delta W = \int d^3x\, \vec{E}\cdot\delta\vec{D} \right)$$

Linear material $\quad \vec{B} = \mu\vec{H}$

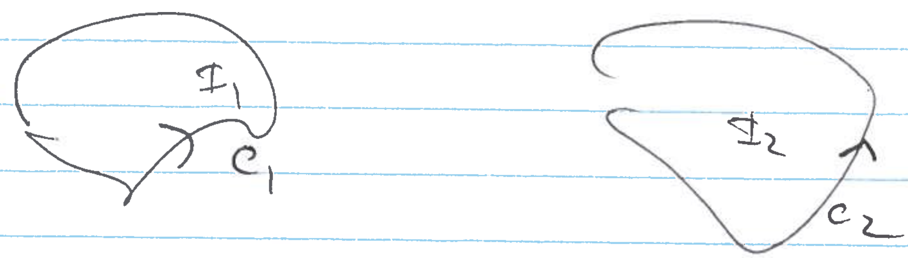
$$\delta(\vec{B}\cdot\vec{H}) = \delta\left(\frac{B^2}{\mu}\right) = \frac{2}{\mu}\vec{B}\cdot\delta\vec{B} = 2\vec{H}\cdot\delta\vec{B}$$

$$\boxed{W = \int d^3x\, \frac{1}{2}\vec{H}\cdot\vec{B} = \int d^3x\, \frac{1}{2}\mu H^2 = \int d^3x\, \frac{1}{2\mu}B^2}$$

Hard magnetization $\vec{M}_0$

$$W = \int d^3x\, \frac{1}{2}\mu_0 \vec{H}^2 - \int d^3x\, \frac{1}{2}\mu_0 \vec{M}_0^2$$

independent of location, orientation




$$\vec{W} = \int d^3x \, \frac{1}{2} \vec{J}(\vec{x}) \cdot \vec{A}(\vec{x})$$

$$= \int d^3x \, \frac{1}{2} \vec{J}(\vec{x}) \cdot \frac{\mu_0}{4\pi} \int d^3x' \, \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|}$$

$$= \sum_{i=1}^{N} \int_{C_i} d^3x_i \, \vec{J}_i(\vec{x}_i) \cdot \frac{\mu_0}{4\pi} \sum_j \int_{C_j} d^3x'_j \, \frac{\vec{J}_j(\vec{x}'_j)}{|\vec{x}_i - \vec{x}'_j|}$$

$$= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} I_i I_j \, \frac{\mu_0}{4\pi} \int_{C_i} \frac{d^3x_i}{I_i} \int_{C_j} \frac{d^3x_j}{I_j} \, \frac{\vec{J}_i(\vec{x}_i) \cdot \vec{J}_j(\vec{x}'_j)}{|\vec{x}_i - \vec{x}'_j|}$$

$$\boxed{W = \frac{1}{2} \sum_i L_i I_i^2 + \sum_{i \neq j} \frac{1}{2} M_{ij} I_i I_j}$$

Self. $$L_i = \frac{\mu_0}{4\pi} \frac{1}{|I_i|^2} \int_{C_i} d^3x'_i \int_{C_i} d^3x'_i \, \frac{\vec{J}_i(\vec{x}_i) \cdot \vec{J}_i(\vec{x}'_i)}{|\vec{x}_i - \vec{x}'_i|}$$

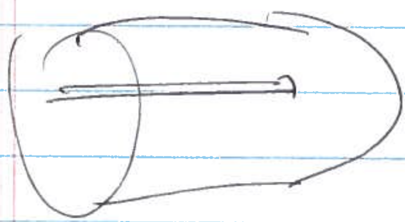
$M_{ij}$ $\quad (i, j)$

## Thin wires

$$M_{ij} = \frac{1}{\cancel{I_i} I_j} \oint_{C_i} \cancel{I_i}\, \vec{dl}_i \cdot \frac{\mu_0}{4\pi} \oint_{C_j} \frac{I_j\, \vec{dl}_j}{|\vec{x}_i - \vec{x}_j|}$$

$$= \frac{1}{I_j} \oint_{C_i} \vec{dl}_i \cdot \vec{A}_j(\vec{x}_i)$$

$$= \frac{1}{I_j} \int_{S_i} d^2a\, \hat{n} \cdot (\vec{\nabla} \times \vec{A}_j) = \frac{1}{I_j} \Phi_{ij} \leftarrow \text{flux from } j \text{ through } i$$



Coax. $\vec{B} = \frac{\mu_0 I}{2\pi\rho} \hat{\phi}$

$$W = \int 2\pi\rho\, d\rho \cdot \ell\, \frac{1}{2\mu_0} \left(\frac{\mu_0 I}{2\pi\rho}\right)^2$$

$$= \frac{1}{2} \frac{\mu_0}{2\pi} I^2 \cdot \ell \int_a^b \frac{\rho\, d\rho}{\rho^2} = \frac{1}{2} \frac{\mu_0 I^2}{2\pi} \ln b/a \cdot \ell$$

$$= \frac{1}{2} L I^2 \qquad \boxed{L/\ell = \frac{\mu_0}{2\pi} \ln b/a}$$

$$W_e = \int 2\pi\rho\, d\rho \cdot \ell \cdot \frac{1}{2} \varepsilon_0 \left(\frac{\lambda}{2\pi\varepsilon_0 \rho}\right)^2 = \frac{1}{2} \frac{1}{2\pi\varepsilon_0} \frac{(\lambda\ell)^2}{\ell} \ln b/a.$$

$$= \frac{Q^2}{2C} \qquad \boxed{C/\ell = \frac{2\pi\varepsilon_0}{\ln b/a}} \qquad \boxed{\frac{C}{\ell} \frac{L}{\ell} = \mu_0 \varepsilon_0 = \frac{1}{c^2}}$$

Need another piece to Maxwell equations

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int d^2a\, \hat{n}\cdot\vec{J} = \mu_0 I \quad \rightarrow \quad \vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n} \qquad \Phi_e = \pi a^2 \sigma/\varepsilon_0 = Q/\varepsilon_0$$

$$\frac{d\Phi_e}{dt} = \frac{I}{\varepsilon_0}. \qquad I_E = \varepsilon_0 \frac{d\Phi}{dt}$$

$$\vec{J}_E = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)}$$

units.

"Displacement current"

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0 = \mu_0\left(\vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)\right)$$

$$= \mu_0\left(\vec{\nabla}\cdot\vec{J} + \varepsilon_0 \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E})\right)$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$

automatic

EM waves relativity. gauge invariance $\leftrightarrow$ renormalizability