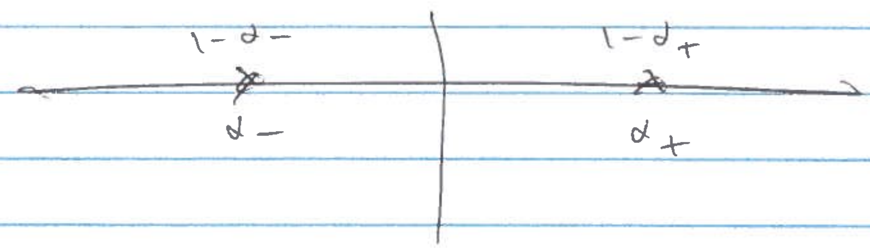
11/30/2015

$$\Box^2 G = \nabla^2 G - \frac{1}{c^2}\frac{\partial^2 G}{\partial t^2} = -4\pi\, \delta^{(3)}(\vec{x}-\vec{x}')\, \delta(t-t')$$

$$G = \int \frac{d^3k}{(2\pi)^3}\frac{d\omega}{2\pi}\, \tilde{G}(\vec{k},\omega)\, e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\left(-|\vec{k}|^2 + \frac{\omega^2}{c^2}\right)\tilde{G} = -4\pi$$

$$G(\vec{x}-\vec{x}', t-t') = \int \frac{d^3k}{(2\pi)^3}\frac{d\omega}{2\pi}\, \frac{-4\pi}{\frac{\omega^2}{c^2}-|\vec{k}|^2}\, e^{i(\vec{k}\cdot(\vec{x}-\vec{x}')-\omega(t-t'))}$$



$\boxed{t-t'>0}$ $$\int d\omega \rightarrow 4\pi i c^2\left(\alpha_+\frac{e^{-ick\Delta t}}{2ck} + \alpha_-\frac{e^{+ick\Delta t}}{-2ck}\right)$$

let $\alpha_+ = \alpha_- = \alpha$

$$G(\Delta\vec{x}, \Delta t) = \int \frac{d^3k}{(2\pi)^3}\, \frac{4\pi i c^2}{2ck}\cdot\alpha\left(e^{-ickt} - e^{+ickt}\right) e^{i\vec{k}\cdot\vec{R}}$$

2

$$\int d\Omega \, e^{ikR\cos\theta} = \int d\phi \int_{-1}^{1} d\mu \, e^{ikr\mu}$$

$$= 2\pi \cdot \left(\frac{e^{ikR} - e^{-ikR}}{ikR} \right) = 4\pi \frac{\sin kR}{kR}.$$

$$\frac{e^{ikR} - e^{-ikR}}{2i}$$

$$G = \frac{\cancel{4\pi} c^2}{(2\pi)^3} \frac{4\pi \alpha}{2c} \int_0^\infty \cancel{k^2} dk \cdot \frac{\sin kR}{\cancel{k}R} \frac{1}{\cancel{k}} \left(e^{-ick\Delta t} - e^{+ick\Delta t} \right)$$

$$= \frac{\alpha \cdot c}{R} \int_0^\infty \frac{dk}{2\pi} \left[e^{ik(R-ct)} - e^{-ik(R+ct)} - e^{ik(R+ct)} + e^{-ik(R+ct)} \right]$$

$$G = \alpha \cdot \frac{c}{R} \left[\delta(R-ct) - \cancel{\delta(R+ct)} \right]$$

argument vanishes only at $|\vec{R}| = 0$ & $t=0$.

$t > t'$ $\quad G_0(\vec{x}, t; \vec{x}', t') = \frac{\alpha}{|\vec{x}-\vec{x}'|} \delta\left(t - t' - \frac{1}{c}R\right)$

retarded

$t < t'$ $\quad G_0 = \frac{(-\alpha)}{|\vec{x}-\vec{x}'|} \delta\left(t - t' + \frac{1}{c}R\right)$

advanced

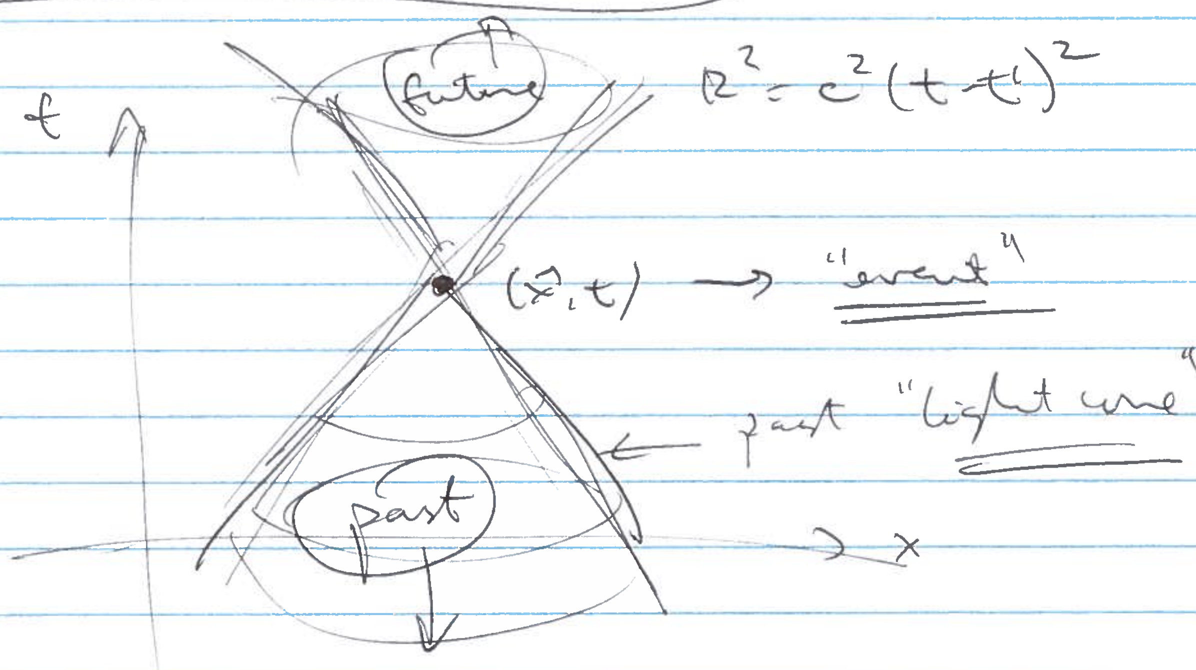
$\alpha = \frac{1}{2}$ Cauchy principal value.

symmetric (Feynman & Wheeler)

Boundary condition "I remember the past, not the future"

$\longleftrightarrow$ "Causality" $\longleftrightarrow$ $\alpha = 1$

$$\boxed{G = \frac{1}{|\vec{x}-\vec{x}'|}\,\delta\left(t - t' - \frac{1}{c}R\right)}$$



(4)

$$\nabla^2 \psi = -4\pi j \rightarrow \psi = \int d^3x' \, dt' \, G(\vec{x}, t, \vec{x}', t') \, j(\vec{x}', t')$$

$$\nabla^2 \Lambda = -\left(\vec{\nabla}\cdot\vec{A} + \mu_0 \varepsilon_0 \frac{\partial \Phi}{\partial t}\right)$$

$$\nabla^2 \vec{A} - \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A} + \mu_0\varepsilon_0 \frac{\partial \Phi}{\partial t}\right) = -\mu_0 \vec{J}$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\nabla^2 \Phi + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) = -\rho/\varepsilon_0 .$$

$$\nabla^2 \Phi = -\rho/\varepsilon_0$$

$$\Phi(\vec{x}, t) = \int d^3x' \, dt' \, \frac{1}{4\pi\varepsilon_0} \frac{\rho(\vec{x}', t')}{|\vec{x} - \vec{x}'|} \, \delta\left(t - t' - \frac{1}{c}|\vec{x} - \vec{x}'|\right).$$

$$\Phi(\vec{x}, t) = \int d^3x' \, \frac{1}{4\pi\varepsilon_0} \frac{\rho(\vec{x}', t_r)}{|\vec{x} - \vec{x}'|} \Bigg|_{t_r = t - \frac{1}{c}|\vec{x} - \vec{x}'|}$$

$\rho(\vec{J})$ independent of t

$$\rightarrow \quad \Phi(\vec{x}) = \int d^3x' \, \frac{1}{4\pi\varepsilon_0} \frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|}$$

$\rho(\vec{J}) \sim e^{-i\omega t}$

$$\Phi(\vec{x}) = \int d^3x' \, \frac{1}{4\pi\varepsilon_0} \frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} e^{i\frac{\omega}{c}|\vec{x}-\vec{x}'|} \quad (e^{-i\omega t})$$

$\leftarrow$ radiation (HW, spring)

Jackson §6.6. lots about materials

$$\vec{\nabla}\cdot\vec{D} = \rho$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J}$$

$\vec{D} = \varepsilon \vec{E}$ linear

$\vec{H} = \frac{1}{\mu}\vec{B}$

$$\frac{1}{\mu_0}\vec{B} - \vec{H} = \vec{M} + (\vec{D} - \varepsilon_0\vec{E})\times\vec{v}$$

(6.100)

But also needs Q_{ij}

$$W = u = \mathcal{E} = \frac{1}{2}\vec{D}\cdot\vec{E} + \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\varepsilon|\vec{E}|^2 + \frac{1}{2\mu}|\vec{B}|^2$$

$$\frac{\partial u}{\partial t} = \vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial \vec{B}}{\partial t}$$

$$= \vec{E}\cdot\left(\vec{\nabla}\times\vec{H} - \vec{J}\right) + \vec{H}\cdot\left(-\vec{\nabla}\times\vec{E}\right)$$

$$= \vec{E}\cdot(\vec{\nabla}\times\vec{H}) - \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{J}\cdot\vec{E}$$

$$\vec{\nabla}\cdot\left(\vec{E}\times\vec{H}\right) = \varepsilon_{ijk}\,\nabla_i E_j H_k$$

$$= \varepsilon_{ijk}(\nabla_i E_j)H_k + \varepsilon_{ijk}E_j(\nabla_i H_k)$$

$$= (\vec{\nabla}\times\vec{E})\cdot\vec{H} - (\vec{\nabla}\times\vec{H})\cdot\vec{E}$$

$$\boxed{\vec{S} = \vec{E}\times\vec{H}} \quad (6.109)$$ Poynting Vector

John Henry Poynting, 1884.
(Statistical Analysis of Commodities Prices).

$$\boxed{\frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} = -\vec{J}\cdot\vec{E}}$$

$\vec{J} = 0$ $\quad \boxed{\frac{\partial u}{\partial t} + \vec{\nabla} \cdot \vec{S} = 0}$ Energy conservation (in field).

$\vec{J} \to$ matter

$\vec{F} = q\vec{E}$: $\frac{d}{dt}\left(\sum \frac{1}{2} m v^2\right) = \sum \left(m \vec{v} \cdot \frac{d\vec{v}}{dt}\right)$

$$= \sum \left(\vec{v} \cdot \vec{F}\right) = \sum \left(\vec{v} \cdot q\vec{E}\right)$$

$$= \sum \left(q\vec{v} \cdot \vec{E}\right) = \vec{J} \cdot \vec{E}$$

$\vec{J} \cdot \vec{E}$ = volume density of rate at which work is done on charged particles

$$E_{Field} = \int d^3x \cdot u$$

$$E_{particles} = \sum \frac{1}{2} m v_i^2 = \int d^3x \, \frac{1}{2} \rho_m v^2$$

$$\frac{d}{dt}\left(E_{Field} + E_{matter}\right) = -\vec{\nabla} \cdot \vec{S}$$

change in $\vec{\ }$: $\longleftrightarrow$ flow in/out of V.