12|2|2015

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ maxwell.

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}$.

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$$ 6.109

$u = \frac{1}{2}\varepsilon_0|\vec{E}|^2 + \frac{1}{2\mu_0}|\vec{B}|^2$.

$$\boxed{\frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} = -\vec{J}\cdot\vec{E}}$$ 6.109

$\frac{d}{dt}\left(E_{field} + E_{particles}\right) = -\oint_S d^2a\, \hat{n}\cdot\vec{S}$

momentum. $\vec{P}_{particles} = \sum p_i$

$\frac{d\vec{P}_{mech}}{dt} = \sum F_i = \sum q_i(\vec{E} + \vec{v}\times\vec{B})$

$\longrightarrow \int d^3x\, \rho(\vec{E} + \vec{v}\times\vec{B}) = \int d^3x\, (\rho\vec{E} + \vec{J}\times\vec{B})$

②

$$\frac{d\vec{P}_{\text{particles}}}{dt} = \int d^3x \left[(\vec{\nabla}\cdot\vec{D})\vec{E} + \left(\vec{\nabla}\times\vec{H} - \frac{\partial\vec{D}}{\partial t}\right)\times\vec{B} \right]$$

$$= \int d^3x \left[\vec{E}(\vec{\nabla}\cdot\vec{D}) + (\vec{\nabla}\times\vec{H})\times\vec{B} - \frac{\partial\vec{D}}{\partial t}\times\vec{B} \right]$$

$$= \int d^3x \left[\varepsilon_0 \vec{E}(\vec{\nabla}\cdot\vec{E}) + \frac{1}{\mu_0}\vec{B}(\vec{\nabla}\cdot\vec{B}) - \frac{1}{\mu_0}\vec{B}\times(\vec{\nabla}\times\vec{B}) + \varepsilon_0 \vec{B}\times\frac{\partial\vec{E}}{\partial t} \right]$$

$$\vec{B}\times\frac{\partial\vec{E}}{\partial t} = -\frac{\partial}{\partial t}(\vec{E}\times\vec{B}) + \vec{E}\times\frac{\partial\vec{B}}{\partial t}$$

$$= -\frac{\partial}{\partial t}(\vec{E}\times\vec{B}) - \vec{E}\times(\vec{\nabla}\times\vec{E})$$

$$\frac{d}{dt}\vec{P}_{\text{mech.}} = -\frac{d}{dt}\int d^3x\, \varepsilon_0 \vec{E}\times\vec{B} \leftarrow \frac{d}{dt}(\vec{P}_{\text{field}}).$$

$$+ \int d^3x \left[\varepsilon_0\left(\vec{E}(\vec{\nabla}\cdot\vec{E}) - \vec{E}\times(\vec{\nabla}\times\vec{E})\right) + \frac{1}{\mu_0}\left(\vec{B}(\vec{\nabla}\cdot\vec{B}) - \vec{B}\times(\vec{\nabla}\times\vec{B})\right) \right]$$

$$\vec{g} = \varepsilon_0 \vec{E}\times\vec{B} = \frac{1}{c^2}\vec{S} =$$ ~~momentum~~ momentum density

③.

$$\vec{E}(\vec{\nabla}\cdot\vec{E}) = E_i(\nabla_j E_j)$$

$$\vec{E}\times(\vec{\nabla}\times\vec{E}) = E_j(\nabla_i E_j) - E_j(\nabla_j E_i)$$

$$\nabla_j\left[E_i E_j - \tfrac{1}{2}\delta_{ij} E^2\right]$$

$$= (\nabla_j E_i) E_j + E_i(\nabla_j E_j) - \tfrac{1}{\cancel{2}}\delta_{ij}\, \cancel{2} E_k \nabla_j E_k$$

$$= E_i(\nabla_j E_j) + E_j(\nabla_j E_i) - E_k(\nabla_i E_k)$$

$$= \vec{E}(\vec{\nabla}\cdot\vec{E}) - \vec{E}\times(\vec{\nabla}\times\vec{E})$$

$$T_{ij} = \varepsilon_0\left(E_i E_j - \tfrac{1}{2}\delta_{ij} E^2\right) + \frac{1}{\mu_0}\left(B_i B_j - \tfrac{1}{2}\delta_{ij} B^2\right)$$

6.120

$$\nabla_j T_{ij} = \varepsilon_0\left[\vec{E}(\vec{\nabla}\cdot\vec{E}) - \vec{E}\times(\vec{\nabla}\times\vec{E})\right.$$

$$\left. + \frac{1}{\mu_0}\left(\vec{B}(\vec{\nabla}\cdot\vec{B}) - \vec{B}\times(\vec{\nabla}\times\vec{B})\right)\right]$$

Maxwell Stress ~~Tensor~~

$$\frac{d}{dt}\left(\vec{P}_{field} + \vec{P}_{mech}\right)_i = \oint_S d^2a\, \hat{n}_j T_{ij}$$

T_{ij} = flux of (p_i)-momentum in $(\hat{n}_j)$ direction

Angular momentum.

$$\vec{L} = \int d^3x \; \vec{x} \times \vec{g} = \int d^3x \; \vec{x} \times \left(\frac{1}{c^2} \vec{E} \times \vec{H} \right)$$

- point e @ origin $\quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{e\hat{r}}{r^2}$

- point g @ $\vec{R}$. $\quad \vec{B} = \frac{1}{4\pi} \frac{g(\vec{x}-\vec{R})}{|\vec{x}-\vec{R}|^3}$

$\vec{\nabla}\cdot\vec{B} = \rho_m$ $\qquad$ $\vec{\nabla}\cdot\vec{H} = \frac{1}{\mu_0}\rho_m$

$$\vec{L} = \int d^3x \; \frac{1}{c^2} \frac{e}{4\pi\varepsilon_0} \frac{\vec{x} \times (\vec{x} \times \vec{H})}{r^3}$$

$$= \frac{1}{c^2} \frac{e}{4\pi\varepsilon_0} \int d^3x \; \frac{1}{r} \; \hat{r} \times (\hat{r} \times \vec{H})$$

$$= \frac{1}{c^2} \frac{e}{4\pi\varepsilon_0} \int d^3x \; \frac{1}{r} \left[\hat{r}(\hat{r}\cdot\vec{H}) - \vec{H} \right]$$

$$(\vec{H}\cdot\vec{\nabla})\hat{r} = H_k \nabla_k \left(\frac{x_i}{r} \right) = H_k \left(\frac{\delta_{ik}}{r} - \frac{x_i x_k}{r^3} \right)$$

$$= \frac{\vec{H}}{r} - \frac{\vec{x}(\vec{x}\cdot\vec{H})}{r^3} = \frac{1}{r}\left[\vec{H} - \hat{r}(\hat{r}\cdot\vec{H}) \right].$$

5

$$\vec{L} = \frac{1}{c^2}\frac{e}{4\pi\varepsilon_0}\int d^3x\,(-\vec{H}\cdot\vec{\nabla})\,\hat{r}$$ ← partly.

$$= \frac{1}{c^2}\cdot\frac{e}{4\pi\varepsilon_0}\int d^3x\left[-\nabla_k(H_k\hat{r}_i) + \hat{r}_i(\nabla_k H_k)\right]$$

$\hookrightarrow \oint d^2a\,\hat{r}_k H_k \hat{r}_i$ → 0

$$= \int_{r\to\infty} r^2 d\Omega\left(\frac{g}{4\pi r^2}\hat{r} + O\left(\frac{1}{r^3}\right)\right)$$

$$\vec{L} = \cancel{\mu_0\varepsilon_0}\cdot\frac{e}{4\pi\cancel{\varepsilon_0}}\int d^3x\,\hat{r}(\vec{\nabla}\cdot\vec{H})$$

$\hookrightarrow \frac{g}{\cancel{\mu_0}}\delta(\vec{x}-\vec{R})$

$$\boxed{\vec{L} = \frac{eg}{4\pi}\hat{R}}$$

Static monopole fields.

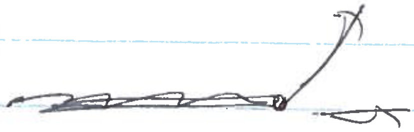
A. Goldhaber, PR 140, B1407 (1965)

(1 year after Ph.D)

$$|\vec{L}| = \frac{eg}{4\pi} = \frac{n}{2}\hbar \qquad \boxed{eg = \frac{1}{2}n\cdot 4\pi\hbar}$$

$$\frac{g\cdot c}{(e/\varepsilon_0)} = \frac{1}{2}\frac{4\pi\hbar c}{e^2/\varepsilon_0} = \frac{1}{2}\left(\frac{e^2/4\pi\varepsilon_0}{\hbar c}\right)^{-1} = \alpha^{-1}/2 = 68.5$$

$$\frac{\varepsilon_0 E^2}{B^2/\mu_0} = \frac{E^2}{c^2B^2} = \frac{(e/\varepsilon_0)^2}{c(g)^2}$$



$$\boxed{\vec{A} = \frac{g}{4\pi}\,\frac{(1-\cos\theta)}{r\sin\theta}\,\hat{\phi}}$$

$g = \pi b^2 \cdot \mu_0 K.$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \hat{r}\,\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\phi) = \frac{g}{4\pi}\frac{\hat{r}}{r^2}$$

$$\cancel{-\hat{\theta}\cdot\frac{1}{r}\frac{\partial}{\partial r}(r A_\phi)}$$

$$H = \frac{1}{2m}(\vec{p}-e\vec{A})^2 + V.$$

Near string $\quad \psi = A\cdot e^{i\alpha}$

$\theta \to \pi \quad 1-\cos\theta \simeq 2$, $\sin\theta \to$ small. $\quad \big|\quad \vec{A}\psi$ big

need: $\vec{p}\psi$ (almost) cancels

$|\psi|^2 = |A|^2$ reasonable $\to \nabla\alpha$ big.

$$\vec{p}\psi = -i\hbar\vec{\nabla}\psi = -i\hbar\left(\cancel{\frac{\partial\psi}{\partial r}\hat{r} + \dots} + \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}\right)$$

$$= -i\hbar\cdot\frac{1}{r\sin\theta}\cdot A\left(i\frac{\partial\alpha}{\partial\phi}\right)e^{i\alpha}$$

$$\hat{\phi}\cdot\vec{\nabla}\psi = \frac{\hbar}{r\sin\theta}\frac{\partial\alpha}{\partial\phi}\psi = \frac{g e(2)}{4\pi\, r\sin\theta}\psi$$

$$\int_0^{2\pi} \hbar \frac{\partial \alpha}{\partial \varphi} = \hbar \cdot \Delta\alpha = \hbar \cdot 2\pi n$$

$$= \int d\ell \cdot \frac{eg}{2\pi} = eg.$$

$$\frac{eg}{4\pi\hbar} = n/2$$