12 | 9 | 2015

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ — Scalar

$\vec{\nabla}\cdot\vec{B} = 0$ — ps.

$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ — pv.

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}$ — vector

$u : \frac{1}{2}\varepsilon_0|\vec{E}|^2 + \frac{1}{2\mu_0}|\vec{B}|^2$

Scalar
P, T, C even

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

vector
P, T odd C even.

(Scalar, vector in another sense.

Let. $\vec{E}' = \vec{E}\cos\xi + c\vec{B}\sin\xi$ (pseudoscalar ξ)

$c\vec{B}' = -\vec{E}\sin\xi + c\vec{B}\cos\xi$

w/o sources : $\vec{\nabla}\cdot\vec{E}' = 0$ $\vec{\nabla}\cdot\vec{B}' = 0$

$$\vec{\nabla}\times\vec{E}' = \cos\xi\,(\vec{\nabla}\times\vec{E}) + \sin\xi\,(\vec{\nabla}\times c\vec{B})$$

$$= \cos\xi\left(-\frac{\partial\vec{B}}{\partial t}\right) + c\cdot\sin\xi\left(\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}\right)$$

$$= -\frac{\partial}{\partial t}\left(\vec{B}\cos\xi - \frac{1}{c}\vec{E}\sin\xi\right) = -\frac{\partial}{\partial t}\vec{B}'$$

$$\vec{\nabla}\times\vec{B}' = \cos\xi\,(\vec{\nabla}\times\vec{B}) - \frac{1}{c}\sin\xi\,(\vec{\nabla}\times\vec{E})$$

$$= \cos\xi\left(\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}\right) - \frac{1}{c}\sin\xi\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$= \frac{1}{c^2}\cdot\frac{\partial}{\partial t}\left(\vec{E}\cos\xi + c\vec{B}\sin\xi\right) = \frac{1}{c^2}\frac{\partial\vec{E}'}{\partial t}$$

Maxwell ✓

$$u' = \frac{1}{2}\varepsilon_0\,\vec{E}'\cdot\vec{E}' + \frac{1}{2\mu_0}\vec{B}'\cdot\vec{B}'$$

$$= \frac{1}{2}\varepsilon_0\left(\vec{E}\cos\xi + c\vec{B}\sin\xi\right)^2 + \frac{1}{2\mu_0}\left(\vec{B}\cos\xi - \frac{1}{c}\vec{E}\sin\xi\right)^2$$

$$= \frac{1}{2}\left(\varepsilon_0|\vec{E}|^2\cos^2\xi + \frac{1}{\mu_0 c^2}|\vec{E}|^2\sin^2\xi\right)$$

$$+ \frac{1}{2}\left(\frac{1}{\mu_0}|\vec{B}|^2\cos^2\xi + \varepsilon_0 c^2|\vec{B}|^2\sin^2\xi\right)$$

$$+ \frac{2}{2}\varepsilon_0 c\,\vec{E}\cdot\vec{B}\sin\xi\cos\xi - \frac{2}{2\mu_0 c}\vec{E}\cdot\vec{B}\cdot s\cdot c.$$

③ -

$$\vec{S}' = \frac{1}{\mu_0}\hat{\vec{E}}' \times \hat{\vec{B}}' = \hat{\vec{S}}.$$

"Duality rotation":

$\xi = \frac{\pi}{2}$: $\vec{E} \to c\vec{B}$

$c\vec{B} \to -\vec{E}$

discrete version:

$p \to q$ $\quad q \to -p$

$$\vec{\nabla}\cdot\vec{D} = \rho_e. \qquad \vec{\nabla}\cdot\vec{B} = \rho_m$$

$$\vec{\nabla}\times\vec{H} - \frac{\partial\vec{D}}{\partial t} = \vec{J}_e \qquad -\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = \vec{J}_m$$

ρ_m would be pseudoscalar.

$\vec{J}_m$ would be pseudovector.

$$\vec{F} = q_e(\vec{E} + \vec{v}\times\vec{B}) + q_m\left(\vec{B} - \frac{\vec{v}\times\vec{E}}{c^2}\right)$$

units

Maxwell , u ; $\vec{S}$, $\cancel{T_{ij}}$.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$

$$W = \frac{1}{2}LI^2 + \frac{1}{2C}Q^2$$

$$\frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} = -\vec{J}\cdot\vec{E}$$

$$\vec{J} = \sigma\vec{E}$$ ← not reversible.

microscopic damping (scattering)

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{E} = -\vec{\nabla}\Phi - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int d^3x' \, \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|}$$

$$\Phi = \frac{1}{4\pi\epsilon_0}\int d^3x' \, \frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|}$$

$$t' = t - \frac{1}{c}R.$$

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial \Phi}{\partial t} = 0$$

6.5 ~~Jefimenko~~ ~~Heaviside-Feynman~~

6.6 molecular ~~quadrupole~~ moment

$$\boxed{\frac{1}{\mu_0}\hat{B} - \vec{H} = \vec{M} + \vec{P} \times \vec{v}} \quad 6.100$$

$$\boxed{\langle AB \rangle = \frac{1}{2} \operatorname{Re}(A^* B)}$$

$$W = q\Phi_0 - \vec{p} \cdot \vec{E}_0 + \cdots \frac{1}{6} Q_{ij}(\partial_i \partial_j \Phi) \ldots$$

$$\vec{F} = q\vec{E}_0 + (\vec{p} \cdot \vec{\nabla})\vec{E}_0 + \cdots$$

$$\boxed{W = -\vec{m} \cdot \vec{B}_0} \leftarrow$$

$$\boxed{\vec{N} = \vec{p} \times \vec{E} = \vec{m} \times \vec{B}}$$

$$eg = 2\pi n \cdot \hbar$$

6.13 ~~Hertz Vectors (Polarization Potentials)~~

$$\langle \vec{E} \rangle_R = -\frac{1}{3\varepsilon_0} \frac{\vec{p}_{in}}{Vol.} + \vec{E}_{out}(0)$$

$$\langle \vec{B} \rangle_R = +\frac{2\mu_0}{3} \frac{\vec{m}_{in}}{Vol.} + \vec{B}_{out}(0)$$

$$\Phi = \sum \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta) \ \ldots$$