While these are some problems that have appeared on previous exams, be aware that they do not cover all possible material.

1. A thin copper ring with radius $r = 10\,\mathrm{cm}$ rotates about an axis perpendicular to a uniform magnetic field $B_0 = 200\,\mathrm{gauss}$ with initial angular frequency ω_0.

(a) As rotational kinetic energy is lost to ohmic heating, show that the rotational frequency decreases exponentially with time, $\omega = \omega_0\, e^{-t/\tau}$, and calculate the time τ it takes the frequency to decrease by a factor of $1/e$ in terms of B_0, r, the conductivity σ, and the mass density ρ. Assume $|d\omega/dt| \ll \omega^2$.

(b) Estimate the decay time numerically. Copper has conductivity $\sigma = 5.8 \times 10^7\,(\Omega \cdot \mathrm{m})^{-1}$ and density $\rho = 8.9\,\mathrm{g\,cm^{-3}} = 8.9 \times 10^3\,\mathrm{kg\,m^{-3}}$.

2. An insulating sphere of radius a rotates slowly with angular frequency ω about the z-axis. Some event has left a charge per unit area $\sigma = \sigma_0 \cos\theta$ fixed on the surface of the sphere.

(a) What is the electric field $\boldsymbol{E}$ inside and outside the sphere?

(b) What is the magnetic field $\boldsymbol{B}$ inside and outside the sphere?

(c) What is the angular momentum contained in the fields?

3. An infinite cylinder carries a uniform surface current $\boldsymbol{K} = K\hat{\boldsymbol{\phi}}$ that wraps around its surface.

(a) Which of the components B_ρ, B_ϕ, B_z can be nonzero? Which of the the coordinates ρ, ϕ, z can the nonzero fields depend on?

(b) Find $\boldsymbol{B}(\boldsymbol{x})$ everywhere.

(c) Use the definition of $\boldsymbol{B}$ in terms of the vector potential $\boldsymbol{A}$ as a sort of Ampère's law to determine $\boldsymbol{A}(\boldsymbol{x})$ inside and outside the cylinder.

(d) If the current increases weakly with time as $dK/dt = \nu K$, what is the electric field inside and outside the cylinder?

(e) If instead of $K\hat{\boldsymbol{\phi}}$, the current consists of a tightly wound coil with N turns per unit length ($Nb \gg 1$), what is $\boldsymbol{B}$ outside the coil?