PHY 6347 Spring 2018

Homework #11, Due Friday April 20

1. A cart rolls on a long table with velocity v. A smaller cart rolls on the first cart in the same direction with velocity v relative to the first cart. A third cart rolls on the second cart in the same direction with relative velocity v, and so on, up to n carts. What is the velocity v_n of the nth cart in the frame of the table? What does v_n tend to as $n \to \infty$? (I can think of at least four ways to do this.)

2. A rocket accelerates from rest to relativistic speed at a rate g experienced in its proper frame. When it is moving at speed v, with Lorentz factor γ, what is the acceleration measured in its original rest frame?

3. A particle of mass m and charge q moves on a circular trajectory in the x-y plane with relativistic speed $\boldsymbol{v} = v\,\hat{\boldsymbol{x}}$ under the influence of a magnetic field $\boldsymbol{B} = B\,\hat{\boldsymbol{z}}$.

(a) What is the force on the particle in the lab frame? What is the acceleration of the particle in the lab frame? What is the radius of its path?

(b) What are the electric and magnetic fields in the instantaneous comoving frame? What is the acceleration of the particle in the instantaneous comoving frame?

4. Let u be the energy density and and $\boldsymbol{S}$ the Poynting vector obtained from electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$.

(a) Write u and $\boldsymbol{S}$ in Gaussian units. Verify that in the absence of charges and currents

$$\frac{\partial u}{\partial t} + \nabla \cdot \boldsymbol{S} = 0$$

for your u and your $\boldsymbol{S}$.

(b) Show that $u^2 - |\boldsymbol{S}|^2/c^2$ is a Lorentz invariant.

(c) Define a velocity $\boldsymbol{v}$ to be $\boldsymbol{v} = \boldsymbol{S}/u$. Write $v = c \tanh 2\zeta$. Compute the Poynting flux $\boldsymbol{S}'$ for fields $\boldsymbol{E}'$, $\boldsymbol{B}'$ obtained from boosting the original fields by velocity $v = c \tanh \zeta$ (half the "rapidity") in the same direction.

5. Ohm's Law in the rest frame says that the current induced by an applied electric field is $\boldsymbol{J} = \sigma \boldsymbol{E}$. Write Ohm's Law in a Lorentz covariant form using the four-vector current density J^μ, the four-velocity u^μ of the conducting material, and the electromagnetic field tensor $F^{\mu\nu}$. Show that for $v \ll c$ this reduces to something sensible.