PHY 6347 Spring 2018

Homework #2, Due Friday, January 26

1. A long coaxial transmission line consisting of two concentric conducting circular cylinders of radii a and b ($a < b$) filled with a lossless insulating dielectric material ($\epsilon = \kappa_e \epsilon_0$, $\mu = \kappa_m \mu_0$) carries a harmonically varying current $I = I_0 \cos \omega t$ along the inner cylinder that returns along the outer cylinder. There are many possible modes, but one has only transverse fields, $E_z = 0$, $B_z = 0$ Formulate this problem with complex fields with the common time dependence $e^{-i\omega t}$.

(a) What are the directions of the electric and magnetic fields for this mode?

(b) From Maxwell's equations, obtain a set of equations relating the field amplitudes E and B.

(c) Combine results of (b) to find the dependence of E, B on z. (Pick the solution that propagates in the $+z$-direction.) Then find E, B in full for $a < \rho < b$.

(d) Find the electric and magnetic energies enclosed in a length ℓ of the cable.

(e) Find the capacitance and inductance per unit length. What is the product $\dfrac{dL}{d\ell}\dfrac{dC}{d\ell}$?

(f) Find the magnitude and direction of the time-average Poynting flux within the interior. Find the net time-average power propagating along the axis. What is the ratio of the power to the electromagnetic energy contained per unit length?

(g) Suppose the conductors are not perfect, but have a finite conductivity σ. The magnetic field then penetrates into the walls of the conductor (the skin depth effect), and the electric field has a (small) component along the surface. What is the rate at which power is lost into the walls? What is the resistance per unit length $\dfrac{dR}{d\ell}$?

(h) Show that the transmitted power is attenuated along the line as $P(z) = P_0 \, e^{-\gamma z}$, and find γ.