1. A lossy material with permeability $\mu = \mu_0$ has permittivity $\varepsilon/\varepsilon_0 = \kappa_1 + i\kappa_2$.

(a) The index of refraction $n^2 = \varepsilon/\varepsilon_0$ is then also complex, $n = n_1 + in_2$. Find $n_1$ and $n_2$ in terms of $\kappa_1$ and $\kappa_2$.

(b) Silver has conductivity $\sigma = 6.30 \times 10^7 (\Omega \text{m})^{-1}$. Find $n_1$ and $n_2$ in the Drude model (take $f_0 = 1$, keep all contributions) for 1 GHz microwaves and for optical frequency. Take the dielectric constant from higher modes to be $\kappa = 2$.

(c) Find the reflection coefficient for 1 GHz microwaves and optical light incident on silver. (Hint: Do not do a new calculation, but apply a known result.)

2. A linearly polarized electromagnetic wave of frequency $\omega$ travels normally through a set of parallel slabs of different index of refraction. Both left- and right-moving waves can appear, and within a given slab the electric field is

$$E_i = E^+_i e^{+ik_i x} + E^-_i e^{-ik_i x},$$

where $k_i^2 = n_i^2 \omega^2/c^2$. The $\pm$ amplitudes can be expressed as a vector, and since Maxwell theory is linear, the effects of crossing a boundary or propagating across a slab can be represented by matrix multiplication $E' = TE$, where

$$E = \begin{pmatrix} E^+ \\ E^- \end{pmatrix}, \quad T = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}.$$

(a) Find the transfer matrix for crossing the boundary from slab $i$ with index $n_i$ to slab $i+1$ with index $n_{i+1}$. Show that $T$ can be written $T = T_0 \tau_0 + T_1 \tau_1$, where $\tau_0 = 1$ is the $2 \times 2$ identity matrix and $\tau_1$ is another name for the usual Pauli matrix.

(b) Find the transfer matrix $T$ that expresses the effect on $E$ of propagating across the thickness $d$ from left to right within slab $i$. Show that $T$ can be written as $T = T_0 \tau_0 + T_3 \tau_3$, where $\tau_3$ is the usual Pauli matrix.

(c) The product of all the interface and slab crossings is a net transfer matrix $T = T_1 T_2 \cdots$ that relates the initial and final amplitudes $E_i$ and $E'_f$. If there is no left-moving wave in the final medium, $E'^-_f = 0$, write the reflected amplitude in the initial medium, $E_{\text{refl}} = E^-_i$, and the transmitted amplitude in the final medium, $E'_{\text{trans}} = E'^+_f$, in terms of the elements of $T$. 
3. An electromagnetic wave of frequency $\omega$ incident from a semi-infinite medium with index of refraction $n_0$ passes through a slab of thickness $d$ with index of refraction $n'$ into a second semi-infinite medium with index of refraction $n_1$.

(a) Use the transfer matrices found in Problem 2 to find the amplitudes of the reflected and transmitted waves.

(b) Show that $n'$ and $d$ can be chosen so that there is no reflected wave. What is the transmitted amplitude for these parameters?

4. An electromagnetic wave of frequency $\omega$ is incident from vacuum through a conducting slab of thickness $d$ with permeability $\mu_c = \mu_0$, permittivity $\epsilon = \kappa \epsilon_0$, and conductivity $\sigma$ and back into vacuum.

(a) Use the transfer matrices found in Problem 2 to find the amplitudes of the reflected and transmitted waves and the reflection and transmission coefficients. It may or may not be useful to write your answer in terms of the small parameters $\eta = (n_r + i n_i)^{-1}$ and $\xi = e^{i(n_r + i n_i)\omega d/c}$.

(b) What is the transmitted amplitude as $d$ becomes large? What is the reflected amplitude as $d$ becomes large?