PHY 6347 Spring 2018

Homework #4, Due Friday, February 9

1. A lossy material with permeability $\mu = \mu_0$ has permittivity $\epsilon/\epsilon_0 = \kappa_1 + i\kappa_2$.

(a) The index of refraction $n^2 = \epsilon/\epsilon_0$ is then also complex, $n = n_1 + in_2$. Find n_1 and n_2 in terms of κ_1 and κ_2.

(b) Silver has conductivity $\sigma = 6.30 \times 10^7\,(\Omega\,\mathrm{m})^{-1}$. Find n_1 and n_2 in the Drude model (take $f_0 = 1$, keep all contributions) for 1 GHz microwaves and for optical frequency. Take the dielectric constant from higher modes to be $\kappa = 2$.

(c) Find the reflection coefficient for 1 GHz microwaves and optical light incident on silver. (Hint: Do not do a new calculation, but apply a known result.)

2. A linearly polarized electromagnetic wave of frequency ω travels normally through a set of parallel slabs of different index of refraction. Both left- and right-moving waves can appear, and within a given slab the electric field is

$$E_i = E_i^+\, e^{+ik_i x} + E_i^-\, e^{-ik_i x},$$

where $k_i^2 = n_i^2\,\omega^2/c^2$. The $\pm$ amplitudes can be expressed as a vector, and since Maxwell theory is linear, the effects of crossing a boundary or propagating across a slab can be represented by matrix multiplication $E' = TE$, where

$$E = \begin{pmatrix} E^+ \\ E^- \end{pmatrix}, \qquad T = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}.$$

(a) Find the transfer matrix for crossing the boundary from slab i with index n_i to slab $i+1$ with index n_{i+1}. Show that T can be written $T = T_0\,\boldsymbol{\tau}_0 + T_1\,\boldsymbol{\tau}_1$, where $\boldsymbol{\tau}_0 = \mathbf{1}$ is the 2×2 identity matrix and $\boldsymbol{\tau}_1$ is another name for the usual Pauli matrix.

(b) Find the transfer matrix T that expresses the effect on E of propagating across the thickness d from left to right within slab i. Show that T can be written as $T = T_0\,\boldsymbol{\tau}_0 + T_3\,\boldsymbol{\tau}_3$, where $\boldsymbol{\tau}_3$ is the usual Pauli matrix.

(c) The product of all the interface and slab crossings is a net transfer matrix $T = T_1\,T_2 \cdots$ that relates the initial and final amplitudes E_i and E'_f. If there is no left-moving wave in the final medium, $E_f'^- = 0$, write the reflected amplitude in the initial medium, $E_{\mathrm{refl}} = E_i^-$, and the transmitted amplitude in the final medium, $E'_{\mathrm{trans}} = E_f'^+$, in terms of the elements of T.

3. An electromagnetic wave of frequency ω incident from a semi-infinite medium with index of refraction n_0 passes through a slab of thickness d with index of refraction n' into a second semi-infinite medium with index of refraction n_1.

(a) Use the transfer matrices found in Problem 2 to find the amplitudes of the reflected and transmitted waves.

(b) Show that n' and d can be chosen so that there is no reflected wave. What is the transmitted amplitude for these parameters?

4. An electromagnetic wave of frequency ω is incident from vacuum through a conducting slab of thickness d with permeability $\mu_c = \mu_0$, permittivity $\epsilon = \kappa\,\epsilon_0$, and conductivity σ and back into vacuum.

(a) Use the transfer matrices found in Problem 2 to find the amplitudes of the reflected and transmitted waves and the reflection and transmission coefficients. It may or may not be useful to write your answer in terms of the small parameters $\eta = (n_r + in_i)^{-1}$ and $\xi = e^{i(n_r+in_i)\omega d/c}$,

(b) What is the transmitted amplitude as d becomes large? What is the reflected amplitude as d becomes large?