PHY 6347 Spring 2018

Homework #6, Due Friday February 23

1. The directions of the real and imaginary parts of the electric dipole moment of an oscillating charge distribution define a plane, and if we take this to be the x-y plane the general electric dipole moment can be written

$$\boldsymbol{p} = p\,(\cos\tfrac{1}{2}\alpha\; e^{-i\beta/2}\,\hat{\boldsymbol{x}} + \sin\tfrac{1}{2}\alpha\; e^{i\beta/2}\,\hat{\boldsymbol{y}}).$$

What is the angular distribution of radiated power for this dipole moment? It is easy to write the result in a way that is difficult to digest. As a step in the right direction, express your result in terms of $\cos\alpha$, $\sin\alpha$, and $\cos\beta$ instead of the half-angles $\frac{1}{2}\alpha$, $\frac{1}{2}\beta$. For what α, β is the result independent of ϕ?

2. Find the angular distribution for quadrupole radiation with the following forms for Q_{ij}:

$$(a)\; Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (b)\; Q = \begin{pmatrix} 0 & i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (c)\; Q = \begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3. A circular loop of radius a centered at the origin in the x-y plane carries an oscillating current $I = I_0 \cos\omega t$.

(a) Calculate the magnetic and electric fields far from the loop ($r \gg a$) but otherwise exactly.

(b) Calculate the power radiated per solid angle. Plot the angular distribution in polar coordinates for $ka = \frac{\pi}{2}$ and $ka = \frac{\pi}{4}$, once for constant current I_0 and once for constant intensity in the equatorial plane (these are two different normalizations). If you feel strong enough, include also $ka = \frac{\pi}{8}$.

(c) Compute (perhaps numerically) the total power radiated for the cases considered in part (b). Use the power to define a "radiation resistance."

(d) Compare the exact result with the leading term in the multipole expansion.

4. A radiation source consists of a square of side a located in the x-y plane and centered at the origin with charges $\pm q$ at alternate corners. The square rotates slowly with angular velocity ω about the z-axis. (What determines how slow is "slow"?)

(a) Find the instantaneous electric dipole moment, magnetic dipole moment, and electric quadrupole moment of this configuration. What is the leading order radiation term? At what frequency is radiation emitted?

(b) Plot the angular distribution of power. Calculate the total radiated power.

5. Rank order the gravitational radiation power emitted by the solar system planets.