PHY 6347 Spring 2018

Homework #9, Due Friday, March 30

1. A conducting sphere of radius a scatters and also absorbs unpolarized incident radiation with wavelength large compared to a. The sphere has conductivity σ such that the skin depth is small compared to a.

(a) Find the exterior magnetic field at the surface of the sphere in the limit that it is an ideal conductor. Show that within the interior of the conductor the field is approximately

$$\boldsymbol{H} \approx \boldsymbol{H}_{\|,0}\, e^{i\xi/\delta}\, e^{-\xi/\delta},$$

where ξ is distance normal to the surface and δ is the skin depth [recall Section 5.18.A]. Show that there is an electric field inside the conductor

$$\boldsymbol{E} \approx \frac{1-i}{\sigma\delta}\,(\hat{\boldsymbol{n}} \times \boldsymbol{H}_{\|,0})\, e^{i\xi/\delta}\, e^{-\xi/\delta},$$

where $\hat{\boldsymbol{n}}$ is directed out of the material.

(b) Compute the flux of power into the material,

$$\frac{dP_{\text{loss}}}{da} = \frac{1}{2\sigma\delta}\,|\hat{\boldsymbol{n}} \times \boldsymbol{H}_0|^2.$$

Integrate to compute the absorption cross section. Write your answer in terms of $k\delta$.

(c) Show that the sphere has a surface impedance

$$\frac{Z_s}{Z_0} = \frac{1}{2}\,k\delta\,(1-i)\,.$$

Show that requiring $E_\|$ to be small compared to incident E_0 is equivalent to $k\delta \ll 1$. Find the coefficients α_l, β_l up to order $(ka)^6$ for $ka \ll 1$ with $Z_s/Z_0 = \frac{1}{2}\,ka(1-i)\delta/a$.

(d) Evaluate the scattering cross section to next-to-leading order in ka. Show that for $\delta/a \to 0$ the cross section reduces to the ideal conductor result.

(e) Evaluate the absorption cross section for small ka.

(f) Find the total cross section from (10.62). Compare with the absorption cross section found in (e). Find where the scattering cross section first appears (whether α_l or β_l, for what value of l, expanded to what order in ka).

2. A uniform, solid sphere of radius a with permittivity $\epsilon = \kappa\,\epsilon_0$, permeability $\mu = \mu_0$, and conductivity σ, scatters circularly polarized incident radiation with wavelength neither large nor small compared to a.

(a) Find the multipole amplitudes α_l, β_l.

(b) Evaluate the scattering and absorption cross sections from (10.61) to leading order and next-to-leading order for small ka. (Take κ to be not too large.)

(c) Find the leading and next-to-leading contributions to the total cross section from (10.62).

(d) Find the scattering cross section for large ka. (Take κ to be not too large.)