1/17/2018

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (-\vec{J}_m)$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t} \qquad (\rho_m)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0 = \mu_0\vec{\nabla}\cdot\vec{J} + \mu_0\varepsilon_0\vec{\nabla}\cdot\frac{\partial \vec{E}}{\partial t}$$

$$\varepsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E}) + \vec{\nabla}\cdot\vec{J} = \boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0}$$

continuity

$$\vec{\nabla}\cdot\vec{B} = 0 \rightarrow \boxed{\vec{B} = \vec{\nabla}\times\vec{A}}$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{\nabla}\times\left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right)$$

$$\rightarrow \boxed{\vec{E} = -\vec{\nabla}\Phi - \frac{\partial \vec{A}}{\partial t}}$$

$$\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\cdot\left(-\vec{\nabla}\Phi - \frac{\partial \vec{A}}{\partial t}\right) = \rho/\varepsilon_0$$

$$\boxed{\nabla^2\Phi + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) = -\rho/\varepsilon_0}$$

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$$

$$= \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial}{\partial t}\left(-\nabla\Phi - \frac{\partial\vec{A}}{\partial t}\right).$$

$$\boxed{\underbrace{\nabla^2\vec{A} - \mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2}}_{\text{wave operator}} - \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A} + \mu_0\varepsilon_0\frac{\partial\Phi}{\partial t}\right) = -\mu_0\vec{J}}$$

wave operator, $\square^2 = \square =$ D'Alembertian.

travelling wave $F(x-ct)$

$$\frac{\partial^2 F}{\partial x^2} = F''(x-ct) \qquad \frac{\partial^2 F}{\partial t^2} = (-c)^2 F''(x-ct).$$

$$\boxed{\frac{\partial^2 F}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} = 0} \qquad \mu_0\varepsilon_0 = \frac{1}{c^2}$$

$$\mu_0\varepsilon_0 = \frac{\mu_0}{4\pi}\cdot 4\pi\varepsilon_0 = \frac{(10^{-7})}{(9\times10^9)} = \frac{1}{(3\times10^8\ \mathrm{m/s})^2} = \frac{1}{c^2}$$

$$\underline{c = 2.99792458\times10^8\ \mathrm{m/s}}$$

would be nice if $\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t} = 0$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J} \qquad \square^2\vec{A} = -\mu_0\vec{J}$$

$$\nabla^2\Phi + \frac{\partial}{\partial t}\left(-\frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right) = -\rho/\varepsilon_0 \qquad \square^2\Phi = -\rho/\varepsilon_0.$$

Can we impose $\vec{\nabla}\cdot\vec{A} + \mu_0\epsilon_0 \frac{\partial\Phi}{\partial t} \to 0$

$\underline{\vec{B} = \vec{\nabla}\times\vec{A}}$. $\vec{A}' = \vec{A} + \vec{\nabla}\Lambda$

$$\vec{B}' = \vec{\nabla}\times\vec{A}' = \vec{\nabla}\times\vec{A} + \underbrace{\cancel{\vec{\nabla}\times\vec{\nabla}\Lambda}}_{d^2\Lambda = 0.} = \vec{B}$$

$\underline{\text{want } \Phi'}$ so that $\vec{E}' = -\vec{\nabla}\Phi' - \frac{\partial\vec{A}'}{\partial t}$.

$$= -\vec{\nabla}\Phi' - \frac{\partial\vec{A}}{\partial t} - \frac{\partial}{\partial t}\vec{\nabla}\Lambda$$

$$= -\vec{\nabla}\left(\Phi' + \frac{\partial\Lambda}{\partial t}\right) - \frac{\partial\vec{A}}{\partial t} = \vec{E} = -\vec{\nabla}\Phi - \frac{\partial\vec{A}}{\partial t}$$

$\Phi' + \frac{\partial\Lambda}{\partial t} = \Phi$ $\boxed{\Phi' = \Phi - \frac{\partial\Lambda}{\partial t}}$ $\vec{A}' = \vec{A} + \vec{\nabla}\Lambda$

Any $\Lambda(\vec{x},t)$. gauge transformation

$$\vec{\nabla}\cdot\vec{A}' + \frac{1}{c^2}\frac{\partial\phi'}{\partial t} = \vec{\nabla}\cdot\vec{A} + \nabla^2\Lambda + \frac{1}{c^2}\frac{\partial\Phi}{\partial t} - \frac{1}{c^2}\frac{\partial^2\Lambda}{\partial t^2}$$

want $\underline{= 0}$ $\Longrightarrow$

want $\underline{\nabla^2\Lambda - \frac{1}{c^2}\frac{\partial^2\Lambda}{\partial t^2} = -\left(\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right)}$

$\Box^2\rho = -4\pi j$

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$$\nabla^2 G = -4\pi\, \delta^{(3)}(x - \vec{x}')$$

$$\square^2 G = -4\pi\, \delta^{(3)}(\vec{x} - \vec{x}')\, \delta(t - t')$$

$$\boxed{G_{adv/ret.} = \frac{1}{|\vec{x} - \vec{x}'|}\, \delta\left(t - t' \pm \frac{1}{c}|\vec{x} - \vec{x}'|\right)}$$

can solve. $\square^2 \psi = -4\pi j$

$$\psi(\vec{x}, t) = \int d^3x'\, dt' \cdot G(\vec{x}, \vec{x}', t, t')\, j(\vec{x}', t').$$

$$= \int d^3x' \frac{1}{|\vec{x} - \vec{x}'|}\, j\left(\vec{x}', t - \frac{1}{c}|\vec{x} - \vec{x}'|\right)$$

$$\Phi(\vec{x}, t) = \int d^3x' \frac{1}{4\pi\varepsilon_0} \left.\frac{\rho(\vec{x}', t')}{|\vec{x} - \vec{x}'|}\right|_{t' = t - \frac{1}{c}|\vec{x} - \vec{x}'|}$$

$$\vec{A}(\vec{x}, t) = \int d^3x' \frac{\mu_0}{4\pi} \left.\frac{\vec{J}(\vec{x}', t')}{|\vec{x} - \vec{x}'|}\right|_{t' = t - \frac{1}{c}|\vec{x} - \vec{x}'|}$$

$$w = u = \mathcal{E} = \frac{1}{2}\varepsilon_0 |\vec{E}|^2 + \frac{1}{2\mu_0}|\vec{B}|^2$$

$$\frac{\partial u}{\partial t} = \varepsilon_0 \vec{E}\cdot\frac{\partial \vec{E}}{\partial t} + \frac{1}{\mu_0}\vec{B}\cdot\frac{\partial \vec{B}}{\partial t}$$

$$= \vec{E}\cdot\left(\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} - \vec{J}\right) + \frac{1}{\mu_0}\vec{B}\cdot\left(-\vec{\nabla}\times\vec{E}\right)$$

$$= \frac{1}{\mu_0}\left[\vec{E}\cdot(\vec{\nabla}\times\vec{B}) - \vec{B}\cdot(\vec{\nabla}\times\vec{E})\right] - \vec{J}\cdot\vec{E}$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{B}) = \varepsilon_{ijk}\,\nabla_i(E_j B_k)$$

$$= \varepsilon_{ijk}(\nabla_i E_j)B_k + \varepsilon_{ijk}E_j(\nabla_i B_k)$$

$$= (\vec{\nabla}\times\vec{E})\cdot\vec{B} - (\vec{\nabla}\times\vec{B})\cdot\vec{E}$$

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}} \quad (6.109)$$

"Poynting Vector"

John Henry Poynting, 1884.
(Statistical Analyses of Commodities Prices)

$$\boxed{\frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} = -\vec{J}\cdot\vec{E}}$$