1/19/2018

maxwell

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0$

$\vec{\nabla}\times\vec{B} - \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} = \mu_0\vec{J}$

$u = \frac{1}{2}\varepsilon_0|\vec{E}|^2 + \frac{1}{2\mu_0}|\vec{B}|^2$

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

$\frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} = -\vec{J}\cdot\vec{E}$ 6.109

w/ No matter: $\vec{J}=0$ $\quad \frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} = 0$

field energy conserved.

matter: $\vec{F} = q\vec{E}$

$$\frac{d}{dt}\left(\sum \tfrac{1}{2} m_i v_i^2\right) = \sum \left(m\vec{v}\cdot\frac{d\vec{v}}{dt}\right) \quad = \int d^3x\, \rho\vec{v}\cdot\vec{E}$$

$$= \sum\left(\vec{v}\cdot\vec{F}\right) = \sum \vec{v}\cdot q\vec{E} = \sum (q\vec{v})\cdot\vec{E} = \vec{J}\cdot\vec{E}$$

$\vec{J}\cdot\vec{E}$ = volume density of rate at which work is done on charged particles

$$E_{field} = \int_V d^3x\, u$$

$$E_{particles} = \sum \tfrac{1}{2} m_i v_i^2 = \int d^3x\, \tfrac{1}{2}\rho_m v^2.$$

$$\frac{d}{dt}\left(E_{field} + E_{particles}\right) = -\oint_S \hat{n}\cdot\vec{S}\, da$$

{change inside V} = {flow out (in) through S}

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<u>momentum</u> : $\vec{P}_{\text{particles}} = \sum \vec{p}_i$

$$\frac{d}{dt}\vec{P}_{\text{mech.}} = \sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_i = \sum q_i\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

$$\longrightarrow \int d^3x\, \rho\left(\vec{E} + \vec{v}\times\vec{B}\right) = \int_V d^3x\left(\rho\vec{E} + \vec{J}\times\vec{B}\right)$$

<u>maxwell</u> $\rightarrow$ $$\frac{d\vec{P}_{\text{mech}}}{dt} = \int d^3x\left[\left(\varepsilon_0\vec{\nabla}\cdot\vec{E}\right)\vec{E} + \left(\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} - \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)\times\vec{B}\right]$$

$$= \int d^3x\left[\varepsilon_0\vec{E}\left(\vec{\nabla}\cdot\vec{E}\right) + \frac{1}{\mu_0}\left(\vec{\nabla}\times\vec{B}\right)\times\vec{B} - \varepsilon_0\frac{\partial\vec{E}}{\partial t}\times\vec{B}\right)$$

$$+\frac{1}{\mu_0}\vec{B}\left(\vec{\nabla}\cdot\vec{B}\right) \qquad + \varepsilon_0\vec{B}\times\frac{\partial\vec{E}}{\partial t}$$

$$\vec{B}\times\frac{\partial\vec{E}}{\partial t} = -\frac{\partial}{\partial t}\left(\vec{E}\times\vec{B}\right) + \vec{E}\times\frac{\partial\vec{B}}{\partial t}$$

$$= -\frac{\partial}{\partial t}\left(\vec{E}\times\vec{B}\right) - \vec{E}\times\left(\vec{\nabla}\times\vec{E}\right)$$

$$\frac{d}{dt}\vec{P}_{\text{mech.}} = -\underbrace{\frac{d}{dt}\int d^3x\,\varepsilon_0\vec{E}\times\vec{B}}_{\frac{d\vec{P}_{\text{field}}}{dt}}$$

$$+\int d^3x\left[\varepsilon_0\left(\vec{E}\left(\vec{\nabla}\cdot\vec{E}\right) - \vec{E}\times\left(\vec{\nabla}\times\vec{E}\right)\right) + \frac{1}{\mu_0}\left(\vec{B}\left(\vec{\nabla}\cdot\vec{B}\right) - \vec{B}\times\left(\vec{\nabla}\times\vec{B}\right)\right)\right]$$

flux. $\rightarrow$ surface

$$\vec{g} = \varepsilon_0\vec{E}\times\vec{B} = \mu_0\varepsilon_0\cdot\vec{S} = \frac{1}{c^2}\vec{S}$$ momentum density

(4)

$$\vec{E}(\vec{\nabla}\cdot\vec{E}) = E_i(\nabla_j E_j) \qquad \Sigma$$

$$\vec{E}\times(\vec{\nabla}\times\vec{E}) = \underline{E_j(\nabla_i E_j)} - (E_j\nabla_j)E_i$$

$$\nabla_j\left[E_iE_j - \tfrac{1}{2}\delta_{ij}E^2\right] = \nabla_j\left[E_iE_j - \tfrac{1}{2}\delta_{ij}E_kE_k\right]$$

$$= (\nabla_j E_i)E_j + E_i(\nabla_j E_j) - \delta_{ij}E_k\nabla_j E_k.$$

$$\underline{E_k(\nabla_i E_k)}$$

$$=$$

$$= E_i(\nabla_j E_j) + E_j(\nabla_j E_i) - E_j(\nabla_i E_j)$$

$$= \vec{E}(\vec{\nabla}\cdot\vec{E}) - \vec{E}\times(\vec{\nabla}\times\vec{E})$$

$$T_{ij} = \varepsilon_0\left(E_iE_j - \tfrac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \tfrac{1}{2}\delta_{ij}B^2\right)$$

<u>maxwell stress tensor</u>. (6.120)

$$\nabla_j T_{ij} = \varepsilon\left[\vec{E}(\vec{\nabla}\cdot\vec{E}) - \vec{E}\times(\vec{\nabla}\times\vec{E})\right] + \frac{1}{\mu_0}\left[\vec{B}(\vec{\nabla}\cdot\vec{B}) - \vec{B}\times(\vec{\nabla}\times\vec{B})\right]$$

$$\boxed{\frac{d}{dt}\left(\vec{P}_{field} + \vec{P}_{mech}\right) = \oint d^2a\, \hat{n}_j T_{ij}}$$

T_{ij} = flux of (p_i)-momentum in $(\hat{n}_j)$-direction

diagonal $\frac{dp_i}{dt\,da} = \frac{dF}{da} =$ pressure. off diag. shear

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Angular momentum.

$$\vec{L} = \int d^3x \, \vec{x} \times \vec{g} = \int d^3x \, \vec{x} \times \left(\frac{1}{c^2} \vec{E} \times \vec{H} \right)$$

Ex — point e @ origin $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{e\hat{r}}{r^2}$

— point g @ $\vec{R}$ $\vec{B} = \frac{1}{4\pi} \frac{g(\vec{x} - \vec{R})}{|\vec{x} - \vec{R}|^3}$

$\vec{\nabla} \cdot \vec{B} = \rho_m$ $\vec{\nabla} \cdot \vec{H} = \frac{1}{\mu_0} \rho_m$

$\hat{r} = \vec{x}/r$

$$\vec{L} = \int d^3x \frac{1}{c^2} \frac{e}{4\pi\varepsilon_0} \cancel{\frac{g}{4\pi\mu_0}} \frac{\vec{x} \times (\vec{x} \times \vec{H})}{r^3}$$

$$= \frac{1}{c^2} \frac{e}{4\pi\varepsilon_0} \int d^3x \frac{1}{r} \hat{r} \times (\hat{r} \times \vec{H})$$

$$[\hat{r}(\hat{r} \cdot \vec{H}) - \vec{H}]$$

$$(\vec{H} \cdot \vec{\nabla}) \hat{r} = H_k \nabla_k \left(\frac{x_i}{r} \right) = H_k \left(\frac{\delta_{ik}}{r} - \frac{x_i x_k}{r^3} \right)$$

$$= \frac{H_i}{r} - \frac{x_i (H_k x_k)}{r^3} = \frac{1}{r} [\vec{H} - \hat{r}(\hat{r} \cdot \vec{H})]$$

⑥

H. A. Wilson, Phys. Rev. $\underline{75}$, 309 (1949)

J. J. Thompson, (Elements of the Mathematical Theory of E & M) (1900).

$$\vec{L} = \frac{1}{c^2}\frac{e}{4\pi\varepsilon_0}\int d^3x\,(-\vec{H}\cdot\vec{\nabla})\,\hat{r} \quad \longleftarrow \text{parts}$$

$$= \frac{1}{c^2}\frac{e}{4\pi\varepsilon_0}\int d^3x\left[-\nabla_k(H_k\hat{r}_i) + \hat{r}_i(\nabla_k H_k)\right]$$

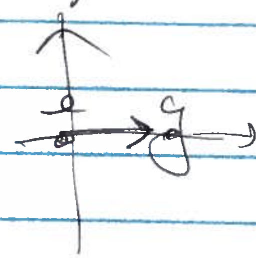
$$\hookrightarrow \oint da\,(\hat{r}_k)\,H_k\hat{r}_k$$

$$= \int_{r\to\infty} r^2 dr\,\hat{r}\left(\cancel{\frac{g\hat{r}}{4\pi r^2}} + O\!\left(\frac{1}{r^3}\right)\right) \to 0$$

$$= \frac{1}{c^2}\frac{e}{4\pi\varepsilon_0}\int d^3x\,(\vec{\nabla}\cdot\vec{H})\,\hat{r}$$

$$\hookrightarrow \frac{g}{\mu_0}\,\delta^{(3)}(\vec{x}-\vec{R})$$

$$\boxed{\vec{L} = \frac{eg}{4\pi}\,\hat{R}}$$



Static fields of e, g.

~~Alfred.~~

A. Goldhaber, Phys. Rev., $\underline{140}$, B1407 (1965) (1 year after Ph.D.).

$$|\vec{L}| = \frac{eg}{4\pi} = \frac{n}{2}\hbar \qquad \boxed{eg = \frac{1}{2}n\cdot 4\pi\hbar}$$

$(n=1)$

$$\frac{gc}{e/\varepsilon_0} = \frac{1}{2}\cdot\frac{4\pi\hbar c}{e^2/\varepsilon_0} = \frac{1}{2}\left(\frac{e^2/4\pi\varepsilon_0}{\hbar c}\right)^{-1} = \frac{1}{2}\alpha^{-1} = \frac{137}{2} = 68.5$$

d'less

$$\frac{\varepsilon_0 E^2}{B^2/\mu_0} = \frac{E^2}{c^2B^2} = \frac{(e/\varepsilon_0)^2}{(cg)^2}$$