1/22/2018.

$$u = \frac{1}{2}\varepsilon_0 |\vec{E}|^2 + \frac{1}{2\mu_0}|\vec{B}|^2$$

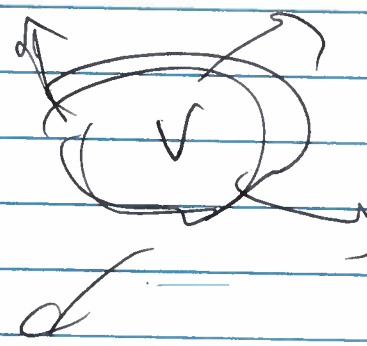
$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \qquad \frac{\partial u}{\partial t} + \hat{\nabla}\cdot\vec{S} = -\vec{J}\cdot\vec{E}$$

$$\vec{g} = \varepsilon_0 \vec{E}\times\vec{B} = \frac{1}{c^2}\vec{S}$$

$$T_{ij} = \varepsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$$

$$\cancel{\tfrac{d}{dt}}\vec{P}_i\Big|_V = \oint_S d^2a\, \hat{n}_j T_{ij}$$

∂B



$$\vec{L} = \varepsilon_0 \int d^3x\ \vec{x}\times\left(\frac{1}{4\pi\varepsilon_0}\frac{e\vec{x}}{r^3} \times \frac{g}{4\pi}\frac{(\vec{x}-\vec{R})}{|\vec{x}-\vec{R}|^3}\right)$$

$$\to \frac{e}{4\pi}\int d^3x\, (\vec{\nabla}\cdot\vec{B})\,\hat{r} = \frac{eg}{4\pi}\hat{R} \qquad = \frac{1}{2}n\hbar$$

A. Goldhaber, PR 140 (1965)
H. A. Wilson, PR 75 (1949)
J. J. Thompson, Elements of the mathematical Theory of E&M. (1900).

$$\frac{c\, g/4\pi r^2}{e/4\pi\varepsilon_0 r^2} = \frac{gc}{e/\varepsilon_0} = \frac{1}{2}n\hbar\frac{4\pi}{e}\cdot\frac{c\varepsilon_0}{e} = \frac{1}{2}n\left(\frac{\hbar c}{e^2/4\pi\varepsilon_0}\right)$$

$$= \frac{1}{2}n\alpha^{-1}.$$

Dirac.

$g = \pi r^2 \cdot \mu_0 k$

$$\boxed{\vec{A} = \frac{g}{4\pi} \frac{(1-\cos\theta)}{r\sin\theta}\hat{\phi}}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{\hat{r}}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{\hat{\theta}}{r}\cancel{\frac{\partial}{\partial r}(rA_\phi)} = \boxed{\frac{g}{4\pi}\frac{\hat{r}}{r^2}}$$

$$\boxed{H = \frac{1}{2m}(\vec{p} - e\vec{A})^2 + V}$$

string should be "invisible" (gauge → move)

near string $\psi = Ae^{i\alpha}$ $\quad \theta \simeq \pi$

$$\vec{A}\psi = \frac{g\,(2)}{4\pi r\sin\theta}\hat{\phi}\, Ae^{i\alpha} \to \text{big} \quad \frac{1}{\sin\theta}$$

need $\vec{p}\psi$ to almost cancel. $|\psi|^2 = |A|^2$ "normal"

$\Rightarrow \nabla\phi$ big. $\quad$ $\theta = \pi - \epsilon$.

$$\hat{\phi}\cdot\vec{p}\psi = (-i\hbar)\left(\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}\right) = -i\hbar\frac{1}{r\sin\theta}\cdot i\frac{\partial\alpha}{\partial\phi}Ae^{i\alpha}$$

$$= \hat{\phi}\cdot e\vec{A} = \frac{eg}{4\pi}\cdot\frac{(2)}{r\sin\theta}Ae^{i\alpha} \qquad \boxed{\frac{\partial\alpha}{\partial\phi} = \frac{eg}{2\pi\hbar}}$$

$$\Delta\alpha = \int d\phi\,\frac{\partial\alpha}{\partial\phi} = \frac{eg}{\hbar} = 2\pi n \qquad \boxed{\frac{eg}{4\pi} = \frac{n\hbar}{2}}$$

unobservable

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Harmonic Time Dependence. $e^{-i\omega t}$ $\quad \mathrm{Re}[\circ]$

[separation of variables, Fourier transform]

$$\vec{E}(\vec{x},t) = \mathrm{Re}\left[\vec{E}(\vec{x})e^{-i\omega t}\right] = \frac{1}{2}\left[\vec{E}(x)e^{-i\omega t} + \vec{E}^*(x)e^{+i\omega t}\right]$$

$\left[\frac{\partial}{\partial t} \to -i\omega\right]$ $\quad (\omega > 0)$.

Energies, fluxes, ... quadratic $\frac{1}{2}\varepsilon_0 |\vec{E}|^2$, $\frac{1}{\mu_0}\vec{E}\times\vec{B}$ (T_{ij}) ...

Time averages. $\langle \cos\omega t\rangle = 0$ $\quad \langle\cos^2\omega t\rangle = \langle\frac{1}{2} + \frac{1}{2}\cancel{\cos 2\omega t}\rangle = \frac{1}{2}$

$= \langle \sin\omega t\rangle = \langle \sin\omega t \cos\omega t\rangle \qquad = \langle\sin^2\omega t\rangle$

$$\langle AB\rangle = \left\langle \frac{1}{2}(A e^{-i\omega t} + A^* e^{+i\omega t}) \cdot \frac{1}{2}(B e^{-i\omega t} + B^* e^{+i\omega t})\right\rangle$$

$$= \frac{1}{4}\langle AB\cancel{e^{-2i\omega t}} + A^*B + AB^* + A^*B^*\cancel{e^{2i\omega t}}\rangle$$

$$= \frac{1}{4}(A^*B + AB^*) = \frac{1}{2}\mathrm{Re}[AB^*] = \frac{1}{2}\mathrm{Re}[A^*B]$$

$\langle\vec{S}\rangle = \frac{1}{2\mu_0}\mathrm{Re}[\vec{E}\times\vec{B}^*]$ $\qquad \vec{S}_{complex} = \frac{1}{2\mu_0}\vec{E}\times\vec{B}^*$

$\qquad\qquad \frac{1}{2}\vec{E}\times\vec{H}^*$

$\langle w_e\rangle = \frac{1}{2}\mathrm{Re}[\vec{E}\cdot\vec{D}^*]$ $\qquad w_e = \frac{1}{4}\vec{E}\cdot\vec{D}^*$

$\langle w_m\rangle = \frac{1}{2}\mathrm{Re}[\vec{B}\cdot\vec{H}^*]$ $\qquad w_m = \frac{1}{4}\vec{B}\cdot\vec{H}^*$

$\langle\vec{J}\cdot\vec{E}\rangle = \frac{1}{2}\mathrm{Re}[\vec{J}\cdot\vec{E}^*]$.

Poynting's Theorem for device

$$\frac{1}{2}\int_V d^3x\, \vec{J}^{*}\cdot\vec{E} = \int_V d^3x \cdot \frac{1}{2}\Big(\vec{\nabla}\times\vec{H} - \underbrace{\frac{\partial \vec{D}}{\partial t}}_{+i\omega\vec{D}}\Big)^{*}\cdot\vec{E}$$

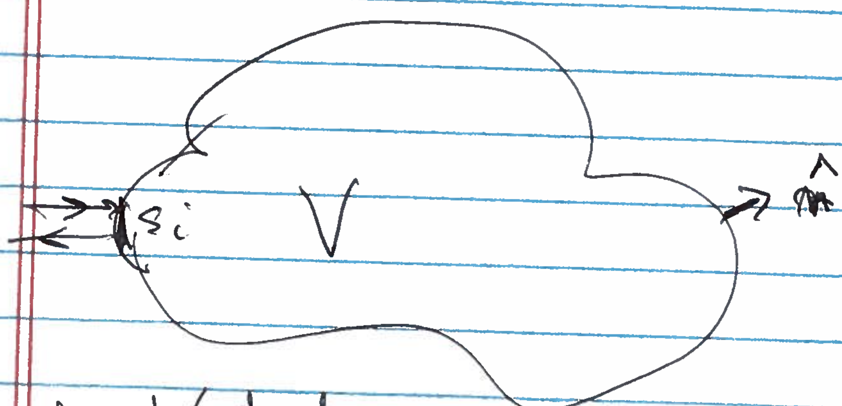
$$= \int_V d^3x\, \frac{1}{2}\left[(\vec{\nabla}\times\vec{H}^{*})\cdot\vec{E} - i\omega\vec{D}^{*}\cdot\vec{E}\right]$$

$$= \int_V d^3x\, \frac{1}{2}\Big[-\vec{\nabla}\cdot(\vec{E}\times\vec{H}^{*}) + \vec{H}^{*}\cdot(\underbrace{\vec{\nabla}\times\vec{E}}_{-\frac{\partial\vec{B}}{\partial t} = +i\omega\vec{B}}) - i\omega\vec{D}^{*}\cdot\vec{E}\Big]$$

$$= \int_V d^3x\, \frac{1}{2}\left[-\vec{\nabla}\cdot(\vec{E}\times\vec{H}^{*}) - i\omega\vec{E}\cdot\vec{D}^{*} + i\omega\vec{B}\cdot\vec{H}^{*}\right]$$

All to LHS:

$$\int_V d^3x\left[\frac{1}{2}\vec{J}^{*}\cdot\vec{E} + 2i\omega(w_e - w_m)\right] + \oint_S da\, \hat{n}\cdot\vec{S} = 0 \qquad (6.134)$$



input/output

current $= I_i$

voltage $= V_i$

$V_i = I_i Z_i$

input impedance

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$$P_i = \int_{S_i} d^2a\,(-\hat{n})\cdot\vec{S} = \frac{1}{2} I_i^* V_i = \frac{1}{2}|I_i|^2 Z_i$$

$$= 2i\omega \int_V d^3x\,(w_e - w_m) + \int_V d^3x\,\frac{1}{2}\vec{J}^*\cdot\vec{E} + \oint_{S-S_i} d^2a\,\hat{n}\cdot\vec{S} \qquad (6.136)$$

Complex: $\boxed{Z = R - iX}$ (E.E.: $e^{+j\omega t}$, $Z = R + jX$)

Real part

$$R = \frac{1}{\frac{1}{2}|I_i|^2}\Big[\mathrm{Re}\int_V d^3x\,\underbrace{\frac{1}{2}\vec{J}^*\cdot\vec{E}}_{\frac{1}{2}\sigma|E|^2} + 2\omega\int d^3x\,\Big[\frac{1}{4}(\mathrm{Im}\,\varepsilon)|E|^2 - \cancel{\frac{1}{4}(\mathrm{Im}\,\mu^{-1})|B|^2}\Big] + \mathrm{Re}\oint_{S-S_i} d^2a\,\hat{n}\cdot\vec{S}\Big]$$

usually negligible

Loss: σ or $(\mathrm{Im}\,\varepsilon)$ (same thing) + R_{rad}

Imag

$$-X = \frac{1}{\frac{1}{2}|I_i|^2}\Big[2\omega\int_V d^3x\,\Big(\frac{1}{4}\mathrm{Re}\,\varepsilon\,|E|^2 - \frac{1}{4}\mathrm{Re}\,\mu^{-1}\,|B|^2\Big) - \cancel{\frac{1}{2}\int_V d^3x\,\mathrm{Im}\,\vec{J}^*\cdot\vec{E}}\Big]$$

usually not.