1/24/2018

$$P_i = \int_{S_i} d^2a \, (-\hat{n}) \cdot \vec{S} = \frac{1}{2} I_i^* V_i = \frac{1}{2} |I_i|^2 Z_i$$

$$= 2i\omega \int_V d^3x \, (w_e - w_m) + \int_V d^3x \, \frac{1}{2} \vec{J}^* \cdot \vec{E} + \oint_{(S-S_i)} d^2a \, \hat{n} \cdot \vec{S}$$

(6.135)

$$\vec{S} = \frac{1}{2\mu} \vec{E} \times \vec{B}^*$$

$$w_e = \frac{1}{4} \varepsilon |E|^2 \qquad w_m = \frac{1}{4\mu} |B|^2 = \frac{1}{4} \mu |H|^2$$

$$Z_i = R - iX$$

$\operatorname{Re} \sigma$

$$R = \operatorname{Re} = \frac{1}{\frac{1}{2}|I_i|^2} \Bigg[ \int_V d^3x \, \frac{1}{2} \sigma |\vec{E}|^2 + 2\omega \int d^3x \, \frac{1}{4} (\operatorname{Im} \varepsilon) |E|^2 + \oint_{S-S_i} d^2a \, \operatorname{Re} (\hat{n} \cdot \vec{S}) \Bigg]$$

$$-X = \operatorname{Im} = \frac{1}{\frac{1}{2}|I_i|^2} \, 2\omega \int_V d^3x \left( \frac{1}{4} \operatorname{Re} \varepsilon |E|^2 - \frac{1}{4} \operatorname{Re} \mu |H|^2 \right)$$

$$R = \frac{1}{|I_i|^2} \int_V d^3x \; \sigma |E|^2$$

$$W_e' - W_m$$

$$X = \frac{4\omega}{|I_i|^2} \int d^3x \cdot \left( \frac{1}{4} \varepsilon |E|^2 - \frac{1}{4} \mu |H|^2 \right)$$

$$W_e = \frac{1}{4} \frac{|Q|^2}{C} = \frac{1}{4} \left| \frac{I}{i\omega} \right|^2 \cdot \frac{1}{C} = \frac{1}{4} \frac{|I|^2}{\omega^2 C}$$

$$W_m = \frac{1}{4} \cdot L \cdot |I|^2$$

$$X = \frac{4\omega}{|I|^2} \left( \frac{1}{4} L |I|^2 - \frac{1}{4} \frac{|I|^2}{\omega^2 C} \right)$$

$$\boxed{X = \omega L - \frac{1}{\omega C}}$$

Resonance $\cdot\ \omega^2 = \frac{1}{LC}$

$$\langle P_{rad} \rangle = \left\langle \oint_S da \, \hat{n} \cdot \vec{S} \right\rangle = \frac{1}{2} |I|^2 R_{rad}$$

Later

$\vec{E}, \vec{B}$ : vector fields.

$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$. scalar.

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, vector.

Q: what is a vector?

- magnitude, direction arrow ↗
- set, closed (addition, scalar multiplication)

  ⊕ , ⊙

Transformation. a vector is something that transforms (rotates) like a vector

rotations preserve lengths, angles

$$\|\vec{x}\| = \sqrt{\vec{x}\cdot\vec{x}} \qquad \cos\theta = \frac{\vec{x}\cdot\vec{y}}{\|\vec{x}\|\,\|\vec{y}\|}.$$

— rotation preserves dot products

$$\vec{x}' = R\vec{x} \qquad \vec{y}' = R\vec{y}$$

$$\vec{x}'\cdot\vec{y}' = (R_{ij}x_j)^T(R_{ik}y_k) = x^T R^T R y$$

$$= x^T(R^T R)y = x^T y. \quad R^T R = 1$$

orthogonal.

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$R^T R = 1$ $\quad (\det R^T)(\det R) = \det \mathbb{1} = 1$

$\det R = \pm 1$ $\quad$ +1: "proper"

derive $\xrightarrow{\text{defer}}$ $\Lambda$.

$v'_i = R_{ij} v_j$ $\quad$ $v' = Rv$. $\quad$ vector

$T'_{ijk} = R_{im} R_{jn} R_{kp} \cdot T_{mnp}$ $\quad$ Tensor (rank = 3).

each shape = vector space. $\quad$ each "leg" rotated

$\cancel{T_{ij} = A_i + B_j}$ wrong

$T_{ij} = A_i B_j$ ✓ outer product. "tensor product."

Special: $(\text{Zero})_{ijk}$ $\quad$ $Z' = RRRZ$ $\quad$ Same in all frames

$\delta'_{ij} = R_{im} R_{jn} \delta_{mn} = (RR^T)_{ij} = \delta_{ij}$

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$\varepsilon'_{ijk} = R_{im} R_{jn} R_{kp} \varepsilon_{mnp}$

$(123) \to \det R$, $\Rightarrow [\det R] (\varepsilon_{ijk})$.

$\boxed{\varepsilon'_{ijk} = \varepsilon_{ijk} \quad \text{for proper } R}$

$\vec{x}' \cdot \vec{y}' = (Rx)^T (Ry) = x^T R^T R y = x^T y$.

scalar.

$T'_{ij} = RR\, T_{ij}$ contraction

$= RR\, (T_{ijkk})$

$=$

$(\vec{x}' \times \vec{y}')_i = \varepsilon_{ijk} x'_j x'_k$

$= \varepsilon_{ijk} \cdot R_{jm} x_m R_{kn} y_n$

$= \delta_{ip} \varepsilon_{pjk} R_{jm} R_{kn} x_m y_n$

$= R_{iq} \boxed{R_{pq} \varepsilon_{ijk} R_{jm} R_{kn}} x_m y_n$

$R_{pq} (\varepsilon'_{qmn} x_m y_n) = R_{pq} (\vec{x} \times \vec{y})_q$

$= R(\vec{x} \times \vec{y})$

vector product is a vector

Dual: $A_i \to \boxed{(*A)_{ij} = \varepsilon_{ijk} A_k}$

$(*A)_{23} = A_1$

$$\begin{pmatrix} 0 & A_3 & -A_2 \\ -A_3 & 0 & A_1 \\ A_2 & -A_1 & 0 \end{pmatrix}$$

Antisymmetric $A_{ij} = -A_{ji}$

$$(*A)_i = \tfrac{1}{2} \varepsilon_{ijk} A_{jk}$$

$$(*A)_1 = \tfrac{1}{2}\left(\varepsilon_{123} A_{23} + \varepsilon_{132} \underbrace{A_{32}}_{=A_{23}}\right) = \tfrac{1}{2}\left(A_{23} - (-A_{23})\right)$$

$*(*A) = A$ $\quad$ $r \leftrightarrow d-r$ $\quad$ $1 \leftrightarrow 2$

$*\phi = \phi \cdot \varepsilon_{ijk}$ $\quad$ $0 \leftrightarrow 3$

$*A_{ijk} = \tfrac{1}{6} \varepsilon_{ijk} A_{ijk}$

In 3d only. Bivector $\leftrightarrow$ vector.

$\vec{x} \times \vec{y} = *(x \wedge y)$

only proper rotations $\rightarrow$ "pseudovector"

$P\vec{x} = -\vec{x}$ $\quad$ $(P\vec{x}) \times (P\vec{y}) = \vec{x} \times \vec{y}$

$\neq P(\vec{x} \times \vec{y})$

vector P-odd

$\vec{x} \cdot \vec{y}$ P-even scalar