1/25/2018

$$T'_{ij} = R_{im} R_{jn} T_{mn}$$

$$= R_{im} T_{mn} R^T_{nj} = R T R^T$$

$$\mathrm{Tr}\, T' = T'_{ii} = R_{im} T_{mn} R^T_{ni}$$

$$= T_{mn} \underline{R^T_{ni} R_{im}} = T_{mn} \delta_{mn} = T_{nn} = \mathrm{Tr}\, T$$

$$T_{ij} = \underline{\tfrac{1}{3} (\mathrm{Tr}\, T) \delta_{ij}} + \underline{\tfrac{1}{2} (T_{ij} - T_{ji})}$$

$$+ \underline{\tfrac{1}{2} (T_{ij} + T_{ji}) - \tfrac{1}{3} (\mathrm{Tr}\, T) \delta_{ij}}$$

scalar . $\mathrm{Tr}\, T' = \mathrm{Tr}\, T$, $\delta' = \delta$ (1)

vector $\underline{V_k} = \cancel{\varepsilon_{ijk} T_{ij}}$ $-\frac{1}{2} \varepsilon_{ijk} T_{jk}$ (3)

"irreducible" $r=2$. symmetric traceless (5)

$E_{ij} = E_{ji}$

$O_{ij} = -O_{ji}$

swap

$$E_{ij} O_{ij} = (+E_{ji})(-O_{ji})$$

$$= E_{j'i'} O_{j'i'}$$

rename.

$$\underline{X = -X} \qquad X + X = -X + X = 0$$

$$= 2X$$

$$E_{ij} O_{ij} = 0 \qquad T_{ij} = T_{(ij)} + T_{[ij]}$$

$$\varepsilon_{ijk} T_{jk} = \varepsilon_{ijk} T_{[jk]}$$

Maxwell:

$\vec{x}$ = prototype vector

$r = |\vec{x}|$ = scalar.

d^3x $\quad \underline{d^3x'} = \det\left|\frac{\partial x'_i}{\partial x_j}\right| d^3x = |\det R| \cdot d^3x = \underline{d^3x}$

Scalar

$\underline{Q}$ is: scalar? tentatively (guess)

$\rho = \frac{dQ}{dV}$ = scalar

$\vec{E} = \int d^3x' \frac{1}{4\pi\epsilon_0} \frac{\rho(\vec{x}')(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}$ vector. P, D

$\Phi = \int d^3x' \frac{1}{4\pi\epsilon_0} \frac{\rho}{|\vec{x}-\vec{x}'|}$ Scalar $\quad \vec{E} = -\vec{\nabla}\Phi$

scalar

electric quantities $\quad$ T-even. $(t \to -t)$

C-odd. $(q \to -q)$

$\vec{J} = \rho\vec{v}$ vector. $\quad$ P-odd

C-odd

T-odd.

$\vec{A} = \int d^3x' \frac{\mu_0}{4\pi} \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|}$ $\quad$ vector

$\vec{B} = \vec{\nabla}\times\vec{A}$ - pseudovector $\quad$ P-even $\quad$ T-odd

C-odd.

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ Scalar P-even C-odd T-even

$\vec{\nabla}\cdot\vec{B} = 0$ pseudoscalar P-odd C-odd T-odd

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ pseudovector P-even C-odd T-even

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$. vector P-odd C-odd T-odd

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$\vec{S} = \vec{E}\times\vec{B}$ vector P-odd C-even T-odd

$u = \frac{1}{2}\varepsilon|E|^2 + \frac{1}{2\mu}|B|^2$ Scalar P-even C-even T-even

$T_{ij} = \varepsilon_0\left(E_iE_j \quad \ldots\right)$ (r=2) Symmetric

P-even C-even T-even.

$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$ vector P-odd. C-even T-even

q^2 $\partial^2/\partial t^2$

④

Chapter 7 E, m. waves.

$\vec{\nabla}\cdot\vec{E} = \cancel{\rho/\epsilon_0} \to 0$

$\vec{\nabla}\cdot\vec{B} = 0$

no sources.

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\vec{\nabla}\times\vec{B} = \cancel{\mu_0\vec{J}} \to 0 + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}$

$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\cancel{\vec{\nabla}\cdot\vec{E}}) - \nabla^2\vec{E}$

$= -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) = -\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$

$\boxed{\nabla^2\vec{E} = \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}}$ $\boxed{\nabla^2\vec{B} = \mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2}}$

$\varepsilon = K_e\varepsilon_0$ $\mu = K_m\mu_0$ $\mu\varepsilon = \frac{K_eK_m}{c^2} = \frac{n^2}{c^2}$

Separation of variables. $\begin{cases}\vec{E} = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{x}-\omega t)} \\ \vec{B} = \vec{B}_0\, e^{i(\vec{k}\cdot\vec{x}-\omega t)}\end{cases}$

$\boxed{|\vec{k}|^2 = \mu\varepsilon\omega^2 = \frac{n^2\omega^2}{c^2}}$ (7.8) $\vec{E}$, $\vec{B}$

$e^{ik(x-\frac{\omega}{k}t)}$

$v_{ph} = \frac{\omega}{|\vec{k}|} = \frac{c}{n}$

$\vec{\nabla}\cdot\vec{E} = i\vec{k}\cdot\vec{E} = 0$ $\quad \vec{k}\cdot\vec{E}_0 = 0$ $\quad \vec{k}\cdot\vec{B}_0 = 0$

Fields $\boxed{\vec{E}_0, \vec{B}_0 \perp \vec{k}}$ "transverse"

$\vec{\nabla}\times\vec{E} = i\vec{k}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = i\omega\vec{B}$

$\boxed{\vec{k}\times\vec{E}_0 = \omega\vec{B}_0}$ write $\vec{k} = \hat{k}k$.

$\boxed{\hat{k}\times\vec{E}_0 = \left(\frac{\omega}{k}\right)\vec{B}_0 = \frac{c}{n}B_0}$

$\hat{E}, \hat{B}, \hat{k} \rightarrow \hat{x}, \hat{y}, \hat{z}$ mutually orthogonal.

Jackson likes $\vec{H}$. $\vec{B} = \mu\vec{H}$

$\vec{\nabla}\times\vec{E} = i\vec{k}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} = +i\mu\omega\vec{H}$

$\hat{k}\times\vec{E}_0 = \frac{\mu\omega}{k}\vec{H} = \frac{\mu}{\sqrt{\mu\epsilon}}\vec{H}_0 = \sqrt{\frac{\mu}{\epsilon}}\vec{H}_0 = Z\vec{H}_0$

$\boxed{\sqrt{\frac{\mu}{\epsilon}} = Z}$ $[E]$ V/m. $\sqrt{\frac{\mu_0}{\epsilon_0}} = Z_0 = 4\pi\sqrt{\frac{\mu_0/4\pi}{\epsilon_0}}$

impedance. $[H]$ A/m

$= 4\pi\sqrt{\frac{\mu_0}{4\pi}}\sqrt{1/4\pi\epsilon_0}$

$= 4\pi\cdot\sqrt{10^{-7}\cdot 9\cdot 10^{9}}$

$= 4\pi\cdot 2.99792458\cdot 10$

$= 376.730313\ \Omega$