1 | 29 | 2018.

$$\vec{E} = \vec{E}_0 \, e^{i(\vec{k}\cdot\hat{x} - \omega t)}$$
$$\vec{B} = \vec{B}_0 \, e^{i(\vec{k}\cdot\hat{x} - \omega t)}$$

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad |\vec{k}|^2 = \mu\varepsilon\,\omega^2 = \frac{n^2 \omega^2}{c^2}$$

$$\mu\varepsilon = \frac{n^2}{c^2}$$

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \vec{\nabla}\cdot\vec{B} = 0 \rightarrow \vec{k}\cdot\vec{E}_0 = 0 \quad \vec{k}\cdot\vec{B}_0 = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{k}\times\vec{E}_0 = \omega \vec{B}_0$$

$$\hat{k}\times\vec{E}_0 = \frac{c}{n}\vec{B}_0$$

Jackson likes $\vec{H}$. $\quad \vec{B} = \mu\vec{H}$

$$\vec{\nabla}\times\vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}. \qquad \hat{k}\times\vec{E}_0 = \frac{\mu\omega}{k}\vec{H}_0$$

$$\frac{\mu\omega}{k} = \mu\cdot\frac{1}{\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} \equiv Z.$$

$[E] = V/m.$

$[H] = [K] = A/m.$

$$\left[\frac{E}{H}\right] = \frac{V}{A} = \Omega.$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = Z_0 = \sqrt{\frac{\mu_0}{4\pi}}\cdot\sqrt{\frac{1}{4\pi\varepsilon_0}}\cdot 4\pi = 4\pi\cdot\sqrt{10^{-7}\cdot 9\cdot 10^9}$$

$$= 4\pi\cdot 10\cdot 2.99792458 = 376.730313 \;\Omega$$

②

$$\langle \vec{S} \rangle = \frac{1}{2} \mathrm{Re}\left(\vec{E} \times \vec{H}^{*} \right)$$

$$= \frac{1}{2} \mathrm{Re}\left(\vec{E}_0 \cancel{e^{i(\vec{k}\cdot\vec{x}-\omega t)}} \times \left(\sqrt{\frac{\varepsilon}{\mu}}\, \hat{k} \times \vec{E}_0 \cancel{e^{i(\vec{k}\cdot\vec{x}-\omega t)}} \right)^{*} \right)$$

$$= \frac{1}{2} \mathrm{Re}\left(\left(\sqrt{\frac{\varepsilon}{\mu}}\right)^{*} \left(\hat{k}\left(\vec{E}_0 \cdot \vec{E}_0^{*}\right) - \vec{E}_0^{*} \cancel{(\hat{k}\cdot\vec{E}_0)} \right) \right)$$

$$\boxed{\langle \vec{S} \rangle = \frac{1}{2} \frac{|\vec{E}_0|^2}{\mathrm{Re}\sqrt{\mu/\varepsilon}}\, \hat{k}}$$

wavevector direction = energy flow direction.

$$[S] = \frac{(V/m)^2}{\Omega} = \frac{(V^2/\Omega)}{m^2} = \frac{\text{Power}}{\text{Area}}$$

$$\langle u \rangle = \mathrm{Re}\left\{ \frac{1}{4} \varepsilon |E|^2 + \frac{1}{4} \mu |H|^2 \right\}$$

$$\hookrightarrow \frac{1}{4} \cancel{\mu} \cdot \frac{\varepsilon}{\cancel{\mu}} \cdot |E|^2$$

$$\boxed{\langle u \rangle = \frac{1}{2} (\mathrm{Re}\,\varepsilon) |\vec{E}_0|^2}$$

$$\frac{\text{rate}}{\text{density}} = \frac{\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}\, |\vec{E}_0|^2}{\frac{1}{2}\varepsilon |E_0|^2} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{n}.$$

ω is real $\rightarrow$ complex ~~n~~ $\rightarrow$ complex $\vec{k}$

$\vec{k} = \vec{k}_r + i\vec{k}_i \rightarrow$ decay $e^{-\vec{k}_i \cdot \vec{x}}$

$$|\vec{k}|^2 = \vec{k} \cdot \vec{k} = k_r^2 - k_i^2 + 2i(\vec{k}_r \cdot \vec{k}_i) = \frac{n^2\omega^2}{c^2}$$

vector

Order of things:

1-wave (dispersion, polarization)

+

Before 2-wave. (reflection / transmission)

(1d). $\hat{k} = \hat{x}$.

$$\psi(x, t_0) = \psi_0(x) = \int \frac{dk}{2\pi} A(k)\, e^{ikx}$$

$$\psi(x,t) = \int \frac{dk}{2\pi} A(k)\, e^{i(kx - \omega t)}$$

$$A(k) = \int dx\, \psi_0(x)\, e^{-ikx}.$$

If: $\frac{\omega}{k} = v$ = constant. $\boxed{\omega = vk}$

$$\psi(x,t) = \int \frac{dk}{2\pi} A(k)\, e^{ik(x-vt)} = \psi_0(x - vt)$$

traveling wave.

@ given ω $e^{i(kx-\omega t)} = e^{ik(x-\frac{\omega}{k}t)} = e^{ik(x-v_{ph}t)}$

$$\boxed{v_{ph} = \frac{\omega}{k}}$$ "phase velocity"

Suppose: $\omega(k)$ not constant +

$A(k)$ peaked near k_0 $\quad \omega(k) \simeq \omega(k_0) + \left(\frac{d\omega}{dk}\right)_0 (k-k_0)$

$$\psi(x,t) = \int dk\, A(k)\, e^{i\left[kx - \left(\omega_0 + \frac{d\omega}{dk}_0 (k-k_0)\right)t\right]}$$

$$= e^{-i\omega_0 t}\, e^{ik_0 \left(\frac{d\omega}{dk}\right)_0 t} \cdot \int dk\, A(k)\, e^{ik\left(x - \left(\frac{d\omega}{dk}\right)_0 t\right)}$$

$$\psi(x,t) = e^{-i(\omega_0 - k_0 v_g)t}\, \psi_0(x - v_g t)$$

$$\boxed{v_{group} = \left.\frac{d\omega}{dk}\right|_{k_0}}$$ "group velocity"

Two waves. $E = \cos(k_1 x - \omega_1 t)\ \cos(k_2 x - \omega_2 t)$

$\bar{k} = \frac{1}{2}(k_1 + k_2) \qquad \bar{\omega} = \frac{1}{2}(\omega_1 + \omega_2)$

$\Delta k = k_1 - k_2 \qquad \Delta\omega = \omega_1 - \omega_2$

$$E = \cos\left((\bar{k} + \tfrac{1}{2}\Delta k)x - (\bar{\omega} + \tfrac{1}{2}\Delta\omega)t\right) + \cos\left((\bar{k} - \tfrac{1}{2}\Delta k)x - (\bar{\omega} - \tfrac{1}{2}\Delta\omega)t\right)$$

$$= 2\cos(\bar{k}x - \bar{\omega}t)\cdot\cos\left(\tfrac{1}{2}\Delta k x - \tfrac{1}{2}\Delta\omega t\right)$$

carrier — modulation term.

$v_{ph} = \frac{\omega}{k} = \frac{c}{n}$. $\quad k = \frac{n\omega}{c}$

$\boxed{v_{ph} = \frac{c}{n}}$

$v_g = \frac{d\omega}{dk} = \frac{1}{dk/d\omega} = \frac{c}{n + \omega\left(\frac{dn}{d\omega}\right)}$.

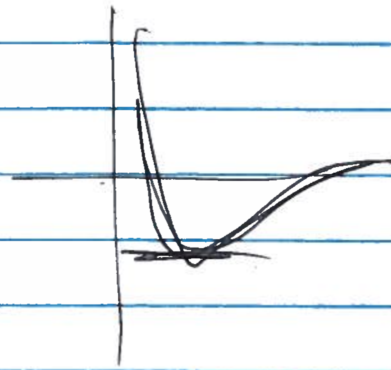
"typically". $\quad n > 1$.

$\frac{dn}{d\omega} > 0$.

$\Leftrightarrow \quad v_g < v_{ph} < c$

$\frac{dn}{d\omega} < 0$. "anomalous dispersion"

Simple ("toy") model.



harmonic oscillator atoms

$$\vec F = -k\vec x - b\vec v + e\vec E$$

$$\uparrow_{m\omega_0^2} \quad \uparrow_{mk.}$$

$$m(\ddot{\vec x} + \gamma\dot{\vec x} + \omega_0^2\vec x) = e\vec E = e\vec E_0\, e^{i(\cancel{\vec k\cdot\vec x}-\omega t)}$$

small

driven: $\vec x = \vec x_0 e^{-i\omega t} + \cancel{(\text{transient})}^{\to 0}$

$$m(-\omega^2 - i\gamma\omega + \omega_0^2)\,\vec x_0 = e\vec E_0.$$

$$\vec x_0 = \frac{e\vec E_0/m}{\omega_0^2-\omega^2-i\gamma\omega} \leftarrow \frac{(\omega_0^2-\omega^2)+i\gamma\omega}{(\omega_0^2-\omega^2)^2+(\gamma\omega)^2}$$

$$\vec p = e\vec x \qquad \vec P = N\cdot\vec p = \frac{Ne^2\vec E_0/m}{\omega_0^2-\omega^2-i\gamma\omega} = \varepsilon_0\chi\vec E$$

$$\frac{\varepsilon}{\varepsilon_0} = 1+\chi = 1 + \frac{N e^2}{m\varepsilon_0}\,\frac{1}{\omega_0^2-\omega^2+i\gamma\omega}$$

multiple modes

$$\boxed{n^2 = \frac{\varepsilon}{\varepsilon_0} = 1 + \frac{Ne^2}{m\varepsilon_0}\sum_k \frac{f_k}{(\omega_k^2-\omega^2)-i\gamma\omega}}$$

$\sum f_k = Z.$ $\qquad$ $NZ = n_e$