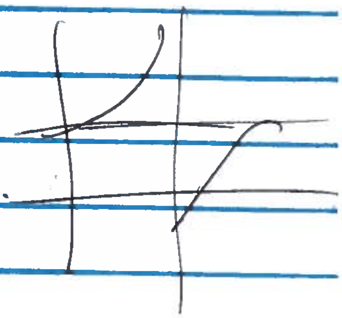
1/31/2008

$v_p = \frac{\omega}{k}$ $\quad$ $v_g = \frac{d\omega}{dk}$

$$\frac{\epsilon}{\epsilon_0} = 1 + \sum_k \frac{Ne^2}{m\epsilon_0} \frac{f_k}{\omega_k^2 - \omega^2 - i\gamma_k\omega}.$$

Free electron gas: $\gamma_k = \omega_k \to 0$.

$$\frac{\epsilon}{\epsilon_0} = 1 - \frac{N_e e^2}{m\epsilon_0\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}.$$



unbound: $\omega_0 \to 0$

$$\frac{\epsilon}{\epsilon_0} = 1 + \sum_{k\neq 0} + \frac{Ne^2}{m\epsilon_0} \frac{1}{(-\omega^2 - i\gamma_0\omega)}$$

$$= K + \frac{Ne^2}{m\epsilon_0} \frac{1}{\omega} \frac{i}{\gamma - i\omega}$$

↑ diverges as $\omega \to 0$.

$$\vec\nabla \times \vec B = \mu\epsilon \frac{\partial \vec E}{\partial t} = (\mu_0)\left(K\epsilon_0 + \frac{ne^2}{m}\frac{i}{\gamma\omega}\right)(-i\omega \vec E)$$

$$= \left(\frac{n\mu_0 e^2}{m\gamma}\right)\vec E - i\omega(\mu\epsilon_0)\vec E$$

$\mu(\sigma\vec E)$ $\quad$ $\vec J = \sigma\vec E$ conductivity.

$\mathrm{Re}\,\epsilon \to K\epsilon_0$

$\mathrm{Im}\,\epsilon \to i\sigma/\omega$

at force level: $\vec{F} = e\vec{E} - \gamma m \vec{v}$

steady state. $m\gamma\vec{v} = e\vec{E}$

$$\vec{J} = ne\vec{v} = \frac{ne^2}{\gamma m}\cdot\vec{E} = \sigma\vec{E}$$

Second order dispersion.

$$\psi(x,t) = \int\frac{dk}{2\pi}A(k)\,e^{i(kx-\omega t)}$$

$$\psi(x,0) = \int\frac{dk}{2\pi}A(k)\,e^{ikx} = \varphi_0(x).$$

$A(k)$ peaked near $\underline{k_0}$. [illegible]

$$\omega(k) = \omega(k_0) + \underbrace{\left(\frac{d\omega}{dk}\right)_0}_{v_g}\Delta k + \frac{1}{2}\underbrace{\left(\frac{d^2\omega}{dk^2}\right)_0}_{\text{dispersion}}\Delta k^2 + \dots$$

near peak. $\ln A = \ln A(k_0) + \cancel{\left(\frac{d\ln A}{dk}\right)_0}\Delta k$

$$+\ \frac{1}{2}\left(\frac{d^2\ln A}{dk^2}\right)_0 \cdot \Delta k^2$$

$\hookrightarrow -L^2$

take: $\boxed{A(k) = A_0\, e^{-\frac{1}{2}L^2(k-k_0)^2}}$

(gaussian → integrals we can do)

$t=0$: $E_0(x) = \int \frac{dk}{2\pi}\, A_0\, e^{-\frac{1}{2}L^2(k-k_0)^2}\, e^{ik_0x}\, e^{i(k-k_0)x}$.

$$= \frac{A_0}{2\pi} e^{ik_0x} \int d(\Delta k)\, \exp\left[-\frac{1}{2}L^2 \Delta k^2 + i\,\Delta k \cdot x\right]$$

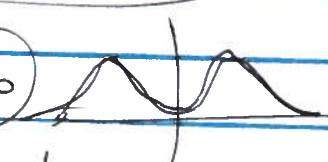
$$= \frac{A_0}{2\pi} e^{ik_0x} \int d\Delta\, \exp\left[-\frac{1}{2}L^2\left(\Delta^2 - \frac{2ix}{L^2}\Delta + \left(\frac{ix}{L^2}\right)^2 - \left(\frac{ix}{L^2}\right)^2\right)\right]$$

$$= \frac{A_0}{2\pi} e^{ik_0x} \cdot \int d\Delta\, \exp\left[-\frac{1}{2}L^2\left(\Delta - \frac{ix}{L}\right)^2 - \frac{1}{2}L^2(-)\left(\frac{ix}{L^2}\right)^2\right]$$

$$\boxed{E_0(x) = \frac{A_0}{2\pi} e^{ik_0x} \cdot \frac{\sqrt{2\pi}}{L} \cdot \exp -\frac{1}{2}\frac{x^2}{L^2}} \quad (7.92)$$

$\cos k_0 x$.

$\cos k_0 x$ peaks @ $\pm k_0$



left-right separation.

$$E(x,t) = \int \frac{dk}{2\pi} A_0 e^{-\frac{1}{2}L^2 (\Delta k)^2} e^{i\left((k_0+\Delta k)x - (\omega_0 + \omega_0' \Delta k + \frac{1}{2}\omega_0'' \Delta^2)t\right)}$$

$$= \frac{A_0}{2\pi} e^{ik_0 x} \int d\Delta \, e^{-\frac{1}{2}L^2\Delta^2} e^{i\Delta x} e^{-i(\omega_0 + \omega_0'\Delta + \frac{1}{2}\omega_0''\delta^2)t}$$

$$= \frac{A}{2\pi} e^{i(k_0 x - \omega_0 t)} \int d\Delta \, \exp\left[i\Delta \cdot (x - \omega_0' t)\right] \times \exp\left(-\frac{1}{2}(L^2 + i\omega_0'' t) \cdot \Delta^2\right)$$

Already done!   center was $x=0$ now $x = v_g t$

width was L^2 now $L^2 + i\omega_0'' t$

$$E(x,t) = \frac{A}{\sqrt{2\pi}\sqrt{L^2 + i\omega_0'' t}} e^{i(k_0 x - \omega_0 t)} \exp\left(-\frac{1}{2}\frac{(x - v_g t)^2}{(L^2 + i\omega_0'' t)}\right)$$

$$\frac{1}{L^2 + i\omega_0'' t} = \frac{L^2 - i\omega_0'' t}{(L^2)^2 + (\omega_0'' t)^2}$$ ← imaginary part → phase

real part → envelope

$$\boxed{(\text{width})^2 = \frac{(L^2)^2 + (\omega_0'' t)^2}{L^2}} = L^2 + \left(\frac{\omega_0'' t}{L^2}\right)^2$$

(7.99)
(7.100)

Polarization (the other kind).

$\vec\nabla\cdot\vec E = 0$. $\vec E_1\cdot\vec E_0 = 0$, $\vec E_{w} = (E_1\vec\varepsilon_1 + E_2\vec\varepsilon_2)\, e^{i(\vec k\cdot\vec x - \omega t)}$

$\hat k = \hat z$ $\vec\varepsilon_1 = \hat x$ $\vec\varepsilon_2 = \hat y$ $\hat k\cdot\vec\varepsilon_i = 0$

$\vec\varepsilon_i^{(*)}\cdot\vec\varepsilon_j = \delta_{ij}$

If it were that simple. ___

Take $E_1 = E_0$ $E_2 = iE_0$ $\vec E = E_0(\vec\varepsilon_1 + i\vec\varepsilon_2)\, e^{i-}$

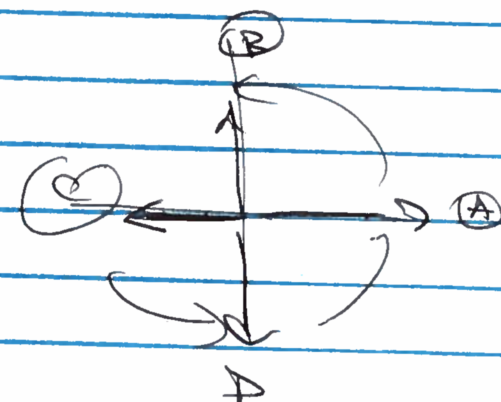
at fixed $\vec x$, $(x=0)$ $\vec E = E_0(\vec\varepsilon_1 + i\vec\varepsilon_2)\, e^{-i\omega t}$

(A) phase = 0 Re → $\vec\varepsilon_1$

(B) phase = $\frac{\pi}{2}$ $e^{-i\frac{\pi}{2}} = -i$ Re → $\vec\varepsilon_2$

(C) phase = π. $e^{-i\pi} = -1$ Re → $-\vec\varepsilon_1$

(D) phase = $\frac{3\pi}{2}$ $e^{-\frac{3\pi i}{2}} = +i$ Re → $-\vec\varepsilon_2$



$\vec E$ sweeps around circle. circular polarization

C6

Any $\vec{E}$ can be expanded in

$$\vec{E} = E_1 \vec{\varepsilon}_1 + E_2 \vec{\varepsilon}_2$$

$$\vec{E} = E_+ \vec{\varepsilon}_+ + E_- \vec{\varepsilon}_-$$

$$\boxed{\vec{\varepsilon}_\pm = \frac{1}{\sqrt{2}}\left(\vec{\varepsilon}_1 \pm i\vec{\varepsilon}_2\right)}$$

$$\vec{\varepsilon}_\pm^{\,*} \cdot \vec{\varepsilon}_\pm = 1 = \vec{\varepsilon}_\pm \cdot \vec{\varepsilon}_\mp$$

$$\vec{\varepsilon}_\pm^{\,*} \cdot \vec{\varepsilon}_\mp = 0 = \vec{\varepsilon}_\pm \cdot \vec{\varepsilon}_\pm$$

$$\boxed{\vec{\varepsilon}_i^{\,*} \cdot \vec{\varepsilon}_j = \delta_{ij}}$$

1,2

±

Arbitrary ("elliptical") polarization

$$\boxed{\vec{\varepsilon} = \cos\frac{\alpha}{2} \cdot e^{-i\beta/2}\,\vec{\varepsilon}_1 + \sin\frac{\alpha}{2}\, e^{+i\beta/2} \cdot \vec{\varepsilon}_2}$$

$\beta = 0$. relatively real. → linear polarization

$\beta = \pm\frac{\pi}{2}$ relatively imaginary. → circular ($\alpha = \frac{\pi}{2}$)

in between: elliptical