2/2/2014 Polarization, $\vec{E}\cdot\vec{k}_0 = 0$. 2-d space

$\vec{\epsilon}_1, \vec{\epsilon}_2$ linear

$\vec{\epsilon}_+, \vec{\epsilon}_- = \frac{1}{\sqrt{2}}(\vec{\epsilon}_1 \pm i\vec{\epsilon}_2)$ circular

either pair: $\vec{\epsilon}_i^{\,*}\cdot\vec{\epsilon}_j = \delta_{ij}$

$E_i = \vec{\epsilon}_i^{\,*}\cdot\vec{E}$ pick off amplitude.

Arbitrary polarization:

$$\vec{\epsilon} = \cos\frac{\alpha}{2}\cdot e^{-i\frac{\beta}{2}}\,\vec{\epsilon}_1 + \sin\frac{\alpha}{2}\, e^{+i\frac{\beta}{2}}\,\vec{\epsilon}_2$$

$\beta=0$ relatively real → linear

$0<\alpha<2\pi$ $\alpha=0$ $\vec{\epsilon}_1$ $\alpha=\frac{\pi}{2}$ $\frac{1}{\sqrt{2}}(\vec{\epsilon}_1+\vec{\epsilon}_2)$ $\alpha=\pi$ $\vec{\epsilon}_2$

$\alpha=\frac{3\pi}{2}$ $\frac{1}{\sqrt{2}}(-\vec{\epsilon}_1+\vec{\epsilon}_2)$ $\alpha=2\pi$ $-\vec{\epsilon}_1$

$0<\beta<\pi$

$\alpha=\frac{\pi}{2}$ $(\pm)$

$\beta=\frac{\pi}{2}$ $(\pm)$ $\frac{1}{\sqrt{2}}(\vec{\epsilon}_1 \pm i\vec{\epsilon}_2)$ circular

Characterize polarization content

"Stokes parameters"

$$\oint_C \vec{A}\cdot d\vec{l} = \int_S (\nabla\times\vec{A})\cdot\hat{n}$$

Navier-Stokes equation

$$\vec{E} \to E_1\vec{\varepsilon}_1 + E_2\vec{\varepsilon}_2 = E_0\cancel{\vec{\varepsilon}_1\cos} + E_0\vec{\varepsilon}$$

$$S_0 = |\vec{\varepsilon}_1^{\,(*)}\cdot\vec{E}|^2 + |\vec{\varepsilon}_2^{\,*}\cdot\vec{E}|^2 = E_1^2 + E_2^2\ \cancel{E_0^2}$$

$$= |E_0|^2\cos^2\frac{\alpha}{2} + |E_0|^2\sin^2\frac{\alpha}{2} = |E_0|^2$$

$$S_1 = |\vec{\varepsilon}_1^{\,*}\cdot\vec{E}|^2 - |\vec{\varepsilon}_2^{\,*}\cdot\vec{E}|^2 = |E_0|^2\cos\alpha$$

$$= E_1^2 - E_2^2 = \underbrace{|E_0|^2\cos^2\frac{\alpha}{2} - |E_0|^2\sin^2\frac{\alpha}{2}}$$

$$S_2 = 2\,\mathrm{Re}\left[(\vec{\varepsilon}_1^{\,*}\cdot\vec{E})^*(\vec{\varepsilon}_2^{\,*}\cdot\vec{E})\right]$$

$$= 2\left[E_0^*\cdot e^{+i\frac{\beta}{2}}\cos\frac{\alpha}{2}\right]\left(E_0\cdot e^{i\frac{\beta}{2}}\sin\frac{\alpha}{2}\right)\Big]$$

$$= \mathrm{Re}\left(|E_0|^2\cdot 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\,e^{i\beta}\right) = |E_0|^2\sin\alpha\cos\beta$$

$$S_3 = 2\,\mathrm{Im}\left[(\vec{\varepsilon}_1^{\,*}\cdot\vec{E})^*(\vec{\varepsilon}_2^{\,*}\cdot\vec{E})\right] = |E_0|^2\sin\alpha\sin\beta$$

$$= \mathrm{Im}\left[|E_0|^2\cdot 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\cdot e^{i\beta}\right)$$

$|\vec{S}|^2 = S_0^2$ completely polarized.

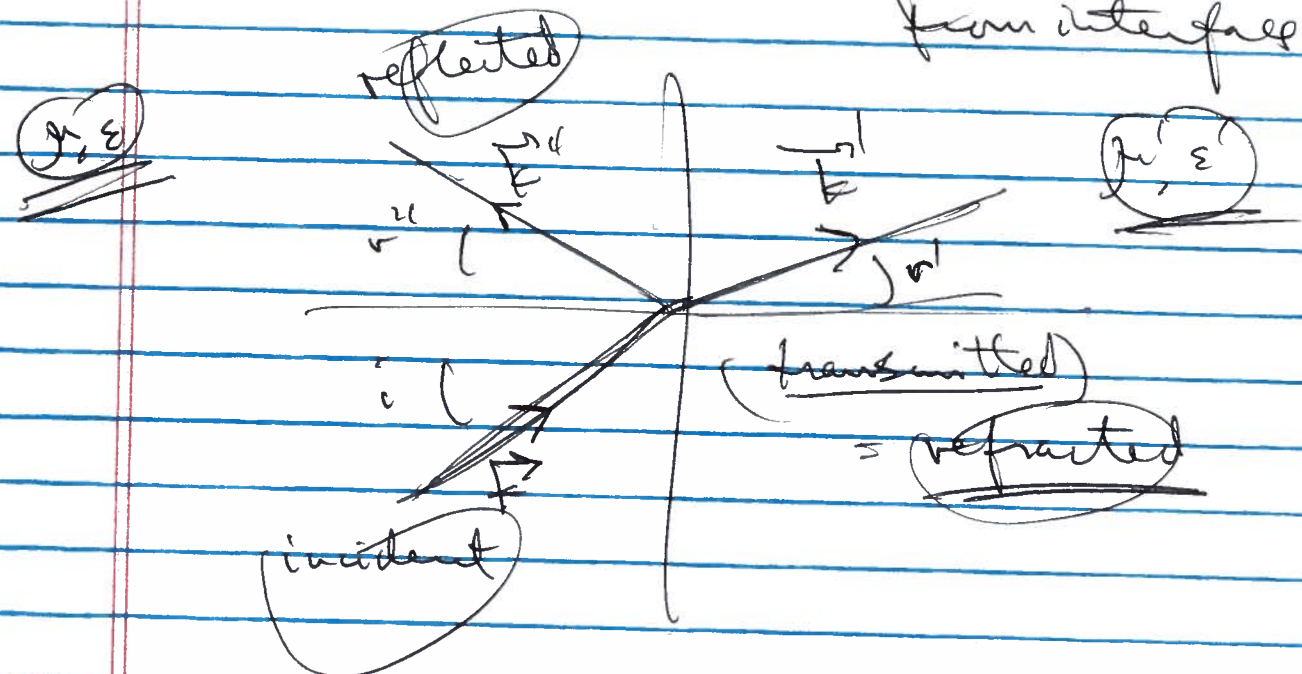
$|\vec{S}|^2 < S_0^2$ partially.

$S_1 = Q = |E_1|^2 - |E_2|^2$

$U = \cancel{\ }$

$V = |E_+|^2 - |E_-|^2$

Two-wave things . reflection from interface



incident $\quad \vec{E} = \vec{E}_0 \, e^{i(\vec{k}\cdot\vec{x} - \omega t)}$

$$\vec{B} = \vec{B}_0 \, e^{i(\vec{k}\cdot\vec{x} - \omega t)} = \sqrt{\mu\varepsilon}\, \hat{k}\times\vec{E} = \frac{n}{c}\,\hat{k}\times\vec{E}$$

reflected $\quad \vec{E}'' = \vec{E}_0'' \, e^{i(\vec{k}''\cdot\vec{x} - \omega t)}$

$$\vec{B}'' = \vec{B}_0'' \, e^{i(\vec{k}''\cdot\vec{x} - \omega t)} = \frac{n}{c}\,\hat{k}''\times\vec{E}''$$

refracted $\quad \vec{B}' = \vec{B}_0' \, e^{i(\vec{k}'\cdot\vec{x} - \omega t)} = \frac{n'}{c}\,\hat{k}'\times\vec{E}'$

$\omega'' = \omega' = \omega$ $\qquad$ No nonlinear optics $\qquad$ red → green at interface ($z=0$) $\vec{x} = \vec{x}_\parallel$ (x, y).

$\vec{k} = \vec{k}_\parallel + k_z\hat{z}$ $\qquad$ $\vec{k}\cdot\vec{x} = \vec{k}_\parallel\cdot\vec{x}_\parallel + (k_z)(0)$

Any kind of matching condition needs same

$$e^{i\vec{k}_\parallel\cdot\vec{x}_\parallel} = e^{i\vec{k}'_\parallel\cdot\vec{x}_\parallel} = e^{i\vec{k}''_\parallel\cdot\vec{x}_\parallel}.$$

all $\vec{x}_\parallel$ $\rightarrow$ $\vec{k}_\parallel = \vec{k}'_\parallel = \vec{k}''_\parallel$

$k_\perp = k\cos\theta$ $\qquad$ $k_\parallel = k\sin\theta$

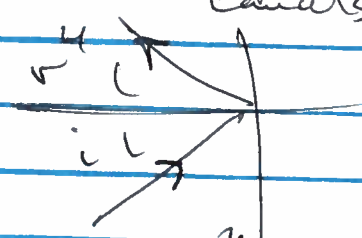
$\Rightarrow$ $k\sin i = k'\sin r' = k''\sin i''$

$$k_z^2 = k^2 - k_\parallel^2 = k^2 - k^2\sin^2\theta = k^2\cos^2\theta = (\pm k\cos\theta)^2$$

$k''_z = \pm k_z$ $\qquad$ $\vec{k}'' = \vec{k}'$ $\rightarrow$ $\vec{E}'' = 0$, $\vec{E}'' = -\vec{E}$

"reflected" cancels incident.

$k''_z = -k_z$



$r'' = i$ $\qquad$ "specular" reflection $\qquad$ (mirror) refracted $k_{\parallel} = k \sin i$

$= k''_{\parallel} = k' \sin r'$

$$\boxed{\frac{n\omega}{c} \sin i = \frac{n'\omega}{c} \sin r'}$$

$$\boxed{n \sin i = n' \sin r'}$$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$ Snell's law

(Any Boundary condition)

Maxwell: $\Delta E_{\parallel} \to 0$ $\Delta D_{\perp} \to 0$ $\Delta B_{\perp} \to 0$ $\Delta H_{\parallel} \to 0$

$$\hat{n} \times (\vec{E}_0 + \vec{E}_0'' - \vec{E}_0') \to 0$$

$$\hat{n} \cdot (\varepsilon(\vec{E}_0 + \vec{E}_0'') - \varepsilon' \vec{E}_0') \to 0$$

$$\vec{\nabla} \times \vec{E} = i\vec{k} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = +i\omega \vec{B}$$

$$\hat{n} \cdot (\vec{k} \times \vec{E}_0 + \vec{k}'' \times \vec{E}_0'' - \vec{k}' \times \vec{E}_0') \to 0$$

$$\hat{n} \times \left(\frac{1}{\mu} \vec{k} \times \vec{E}_0 + \frac{1}{\mu} \vec{k}'' \times \vec{E}_0'' - \frac{1}{\mu'} \vec{k}' \times \vec{E}_0'\right) \to 0$$

6 unknowns $\vec{E}_0'$, $\vec{E}_0''$.

8 equations. $\Rightarrow$ need some check