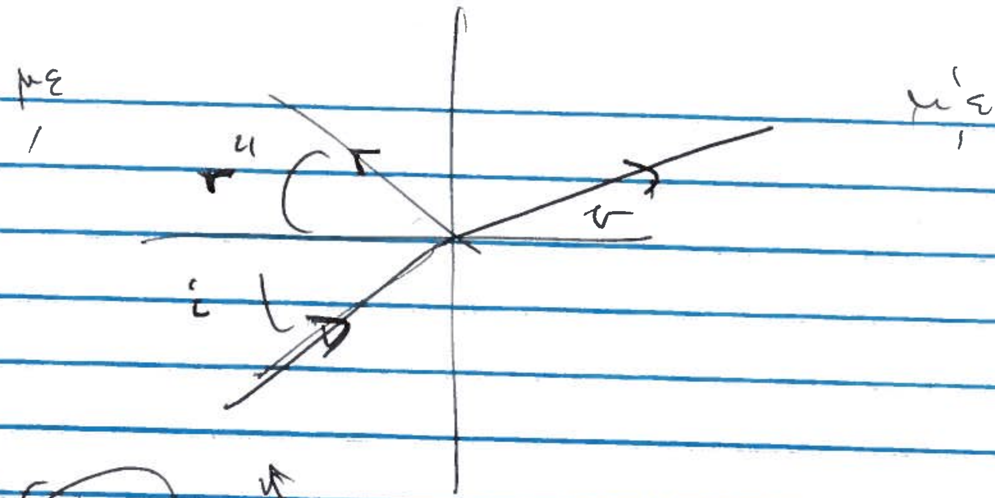
2/5/2018



$r'' = i$ → mirror reflection

$n \sin i = n' \sin r'$ Snell's law.

maxwell $\Delta \vec{E}_\parallel = 0$ $\quad \Delta D_\perp = 0$ $\quad \Delta B_\perp = 0$ $\quad \Delta H_\parallel = 0$

$$\hat{n} \times \left(\vec{E}_0 + \vec{E}_0'' - \vec{E}_0' \right) = 0$$

$$\hat{n} \cdot \left(\varepsilon (\vec{E}_0 + \vec{E}_0'') - \varepsilon' \vec{E}_0' \right) = 0$$

$$\vec{\nabla} \times \vec{E} = i\vec{k} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = +i\omega \vec{B}$$

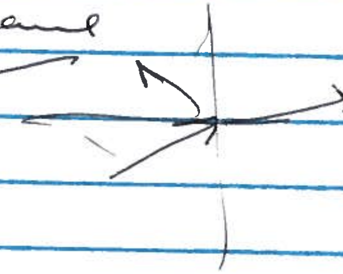
$$\hat{n} \cdot \left(\vec{k} \times \vec{E}_0 + \vec{k}'' \times \vec{E}_0'' - \vec{k}' \times \vec{E}_0' \right) = 0$$

$$\hat{n} \cdot \left(\frac{1}{\mu} \vec{k} \times \vec{E}_0 + \frac{1}{\mu} \vec{k}'' \times \vec{E}_0'' - \frac{1}{\mu'} \vec{k}' \times \vec{E}_0' \right) = 0$$

6 unknowns $\vec{E}_0', \vec{E}_0''$ $\quad$ 8 equations → redundancy.

Geometry let $\hat{n} = \hat{z}$

$\vec{k}, \vec{k}', \vec{k}''$ in x-z plane



$$\vec{k} = k\cos i\,\hat{z} + k\sin i\,\hat{x}$$

$$\vec{k}' = k'\cos r'\,\hat{z} + k'\sin r'\,\hat{x}$$

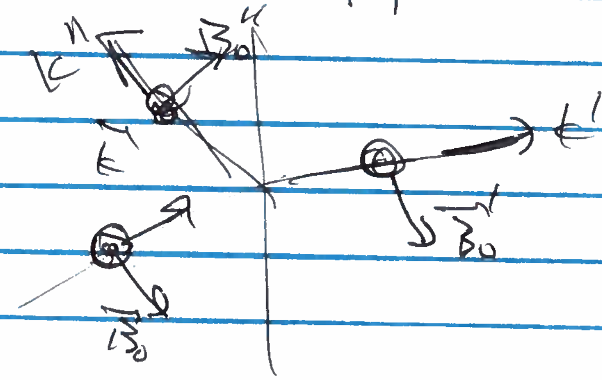
($k\sin i$)

$$\vec{k}'' = -k''\cos r''\,\hat{z} + k''\sin r''\,\hat{x}$$

$$= -k\cos i\,\hat{z} + k\sin i\,\hat{x}''$$

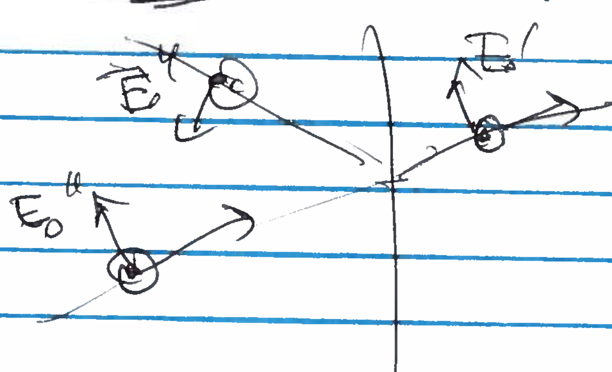
incident $\vec{E}_0$, $\vec{k}\cdot\vec{E}_0 = 0$ 2 possibilities

① ⊥ $\vec{E}_0 \perp$ scattering plane. $\vec{E}_0 = E_0\hat{y}$



② ∥. $\vec{E}_0$ in scattering plane → $\vec{B}_0 \perp$

$\vec{B}_0 = B_0\hat{y}$



$$\vec{\nabla}\times\vec{B} = i\vec{k}\times\vec{B}$$

$$= \mu\epsilon\frac{\partial E}{\partial t} = -i\mu\epsilon\,\vec{k}\times\vec{E}$$

③

① $\hat{z}\cdot\left(\varepsilon(\cancel{E_0}E_0\hat{y}+E_0''\hat{y})-\varepsilon' E_0'\hat{y}\right)=0.$ ✓

$(\Delta D_\perp=0)$.

$$\hat{z}\times\left(E_0+E_0''-E_0'\right)\hat{y} = \left(E_0+E_0'-E_0''\right)(\hat{z}\times\hat{y})=0$$

$$\boxed{E_0+E_0''=E_0'}$$

$\vec{\Delta E_\parallel}=0$

$$\hat{n}\cdot(\vec{k}\times\vec{E}) = (\hat{n}\times\vec{k})\cdot\vec{E} = \left(\hat{z}\times(\cancel{k\cos i\,\hat{z}}+k\sin i\,\hat{x})\right)\cdot\vec{E}$$

$$= k\sin i\,\underline{(\hat{z}\times\hat{x})\cdot E_0\hat{y}} = k\sin i\,E_0$$

$$k\sin i\cdot E_0 + k''\sin r''\cdot E_0'' - k'\sin r'\,E_0' = 0$$

$$\boxed{E_0+E_0''=E_0'}$$

!

$$\hat{n}\times(\vec{k}\times\vec{E}_0) = \vec{k}(\underbrace{\hat{n}\cdot\vec{E}_0}_{\hat{z}\cdot\hat{y}}) - \vec{E}_0(\hat{n}\cdot\vec{k}) = -E_0\hat{y}\cdot(k\cos i)$$

$$\frac{1}{\mu}E_0 k\cos i + \frac{1}{\mu}E_0''(-k\cos i) - \frac{1}{\mu'}E_0'\cdot(k'\cos r')$$

$$\boxed{\frac{n}{\mu}\cos i\,E_0 - \frac{n}{\mu}\cos i\,E_0'' = \frac{n'}{\mu'}\cos r'\,E_0'}$$

$\Delta H_\parallel = 0$

$$n'^2\cos^2 r' = n'^2(1-\sin^2 r') = n'^2 - n'^2\sin^2 r'$$

$$= n'^2 - n^2\sin^2 i$$

4

$$\frac{E_0'}{E_0} = \frac{2n\cos i}{n\cos i' + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\cos^2 i}}$$

$$\frac{E_0''}{E_0} = \frac{n\cos i - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\cos^2 i}}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\cos^2 i}}$$

I

(7.39)

II

$$\frac{n E_0}{\mu} + \frac{n E_0''}{\mu} = \frac{n' E_0'}{\mu'}$$ At $H_\parallel \Rightarrow$.

$$E_0\cos i - E_0''\cos i = E_0'\cos r'$$ At $E_\parallel \Rightarrow$

$$\frac{E_0'}{E_0} = \frac{2nn'\cos i}{\frac{\mu}{\mu'}n'^2\cos i + n\sqrt{n'^2 - n^2\sin^2 i}}$$

$$\frac{E_0''}{E_0} = \frac{\frac{\mu}{\mu'}n'^2\cos i - n\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'}n'^2\cos i + n\sqrt{n'^2 - n^2\sin^2 i}}$$

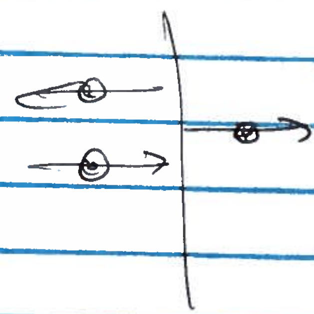
7.41

often. $\mu = \mu' \simeq \mu_0$ (usually? always?)

look at normal incidence. $\sin i \to 1$ ($\cos i \to 0$

I. $\dfrac{E_0'}{E_0} = \dfrac{2n}{n+n'}$

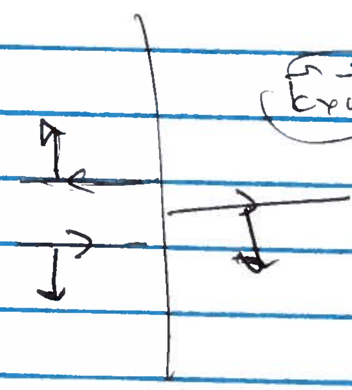
$\dfrac{E_0''}{E_0} = \dfrac{n-n'}{n+n'}$



II. $\dfrac{E_0'}{E_0} = \dfrac{2n}{n+n'}$ ✓

Exps.

$\dfrac{E_0''}{E_0} = \dfrac{n'-n}{n+n'}$?



$n' > n$ = phase reversed

$n' \gg n$. $E_0'' \simeq -E_0$ "ideal" mirror

(conductor → imaginary

→ "more" perfect)

reflection/transmission coefficients →

Power

Transport $\rightarrow \vec{S}$

$$\langle \vec{S}\cdot\hat{n}\rangle = \mathrm{Re}\left[\frac{1}{2}\hat{n}\cdot\left(\vec{E}\times\frac{\vec{B}^*}{\mu}\right)\right]$$

$$= \mathrm{Re}\left[\frac{1}{2}\hat{n}\cdot\left(\vec{E}\times\left(\frac{1}{\mu\omega}\vec{k}\times\vec{E}\right)^*\right)\right]$$

$$= \mathrm{Re}\left(\frac{1}{2\mu\omega}\hat{n}\cdot\left(\vec{k}\,(\vec{E}\cdot\vec{E}^*) - \cancel{\vec{E}^*(\vec{k}\cdot\vec{E})}\right)\right)$$

$$= \frac{\mathrm{Re}\,k}{2\mu\omega}(\hat{n}\cdot\hat{k})\,|E_0|^2$$

$$\frac{k}{\mu\omega} = \frac{\sqrt{\mu\epsilon}}{\mu} = \frac{1}{\sqrt{\mu/\epsilon}} = \frac{n}{Z_0}$$

Normal: $\hat{n}\cdot\hat{k} = 1$

$\cancel{k = \frac{n\omega}{c} = \sqrt{\epsilon'}\,\frac{\omega}{c}} \quad \cancel{\frac{k}{\mu\omega} = \sqrt{\frac{\epsilon}{\epsilon_0}}\,\frac{\sqrt{\mu_0\epsilon_0}}{\mu_0}}$

$$\boxed{P_i = P_0 = \frac{1}{2}\,\mathrm{Re}\,n\,\frac{|E_0|^2}{Z_0}} \qquad P_R'' = \frac{1}{2}\,\mathrm{Re}\,n\,\frac{|E_0''|^2}{Z_0}$$

$$P_T' = \frac{1}{2}\,\mathrm{Re}\,n'\,\frac{|E_0'|^2}{Z_0}$$

$$\boxed{T = \frac{P'}{P_0} = \frac{n'|E_0'|^2}{n|E_0|^2} = \frac{n'\cdot 4n^2}{n(n'+n)^2} = \frac{4nn'}{(n+n')^2}}$$

$$R = \frac{P''}{P_0} = \frac{n|E_0''|^2}{n|E_0|^2} = \frac{(n-n')^2}{(n+n')^2}$$

$$R+T=1.$$