2/7/2018 Transport $\rightarrow \vec{S}$

$$\langle \vec{S}\cdot\hat{n}\rangle = \mathrm{Re}\left[\frac{1}{2}\hat{n}\cdot\left(\vec{E}\times\frac{\vec{B}^*}{\mu}\right)\right]$$

$$= \mathrm{Re}\left[\frac{1}{2}\hat{n}\cdot\left(\vec{E}\times\left(\frac{1}{\mu\omega}\vec{k}\times\vec{E}\right)^*\right)\right]$$

$$= \mathrm{Re}\left(\frac{1}{2\mu\omega}\hat{n}\cdot\left(\vec{k}(\vec{E}\cdot\vec{E}^*) - \vec{E}^*\cancel{(\vec{k}\cdot\vec{E})}\right)\right)$$

$$= \frac{\mathrm{Re}\,k}{2\mu\omega}(\hat{n}\cdot\hat{k})\,|E_0|^2$$

$$\frac{k}{\mu\omega} = \frac{\sqrt{\mu\varepsilon}}{\mu} = \frac{1}{\sqrt{\mu/\varepsilon}} = \frac{n}{Z_0}$$

Normal - $\hat{n}\cdot\hat{k} = 1$

$$k = \frac{n\omega}{c} = \sqrt{\frac{\varepsilon'}{\varepsilon_0}}\frac{\omega}{c} \cdot \frac{k}{\mu\omega} = \sqrt{\frac{\varepsilon}{\varepsilon_0}}\frac{\sqrt{\mu_0\varepsilon_0}}{\mu_0}$$

$$\boxed{P_i = P_0 = \frac{1}{2}\mathrm{Re}(n)\frac{|E_0|^2}{Z_0}} \qquad \boxed{P_R'' = \frac{1}{2}\mathrm{Re}(n)\frac{|E_0''|^2}{Z_0}}$$

$$P_T' = \frac{1}{2}\mathrm{Re}\,n'\frac{|E_0'|^2}{Z_0}$$

$$\boxed{T = \frac{P'}{P_0} = \frac{n'|E_0'|^2}{n|E_0|^2} = \frac{n'\cdot 4n^2}{n(n'+n)^2} = \frac{4nn'}{(n+n')^2}}$$

$$\boxed{R = \frac{P''}{P_0} = \frac{n|E_0''|^2}{n|E_0|^2} = \frac{(n-n')^2}{(n+n')^2}}$$

$$R+T=1.$$

2

more details

⊥ $\frac{E_0''}{E_0} = \frac{n\cos i - \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\sin^2 i}}$

∥ $\frac{E_0''}{E_0} = \frac{\frac{\mu}{\mu'} n'^2\cos i - n\sqrt{n'^2 - n^2\sin^2 i}}{\frac{\mu}{\mu'} n'^2\cos i + n\sqrt{n'^2 - n^2\sin^2 i}}$

TT: $n > n'$ (glass, water → air)

$\sin i$ too big. $\longrightarrow \sqrt{(\text{negative})}$

$k^2 = \frac{n^2\omega^2}{c^2}$ $\quad k'^2 = \frac{n'^2\omega^2}{c^2} \Rightarrow \frac{n'^2\omega^2}{c^2} - \frac{n^2\omega^2\sin^2 i}{c^2}$

$k_z'^2 = \frac{\omega^2}{c^2}(n'^2 - n^2\sin^2 i)$

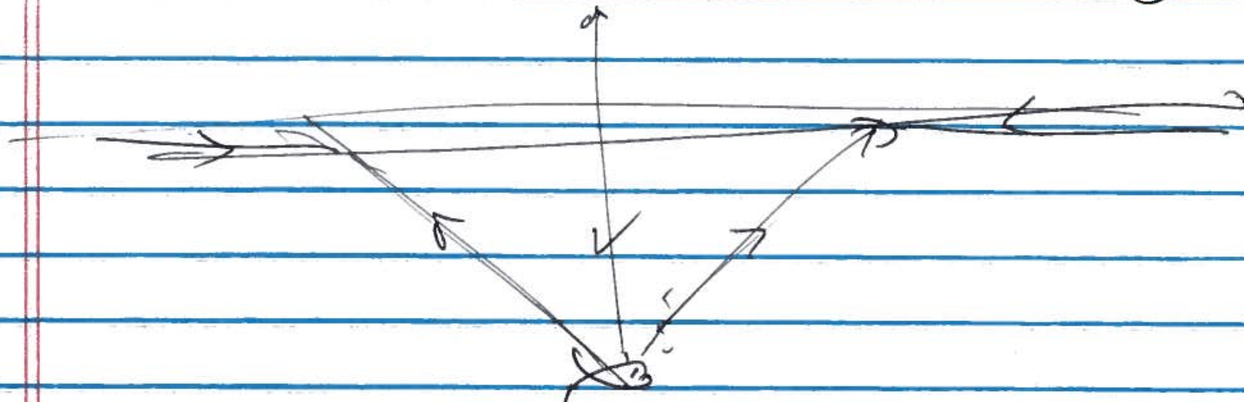
$k_z'^2 < 0$ $\quad k_z'$ imaginary $\quad e^{i\vec{k}\cdot\vec{x}} \to e^{-|k_z'|z}$ $\quad e^{-\kappa z}$

$\langle \vec{S}' \cdot \hat{n} \rangle = \cdots \mathrm{Re}(\hat{n}\cdot\vec{k}') = \mathrm{Re}(ik) = 0.$

T = 0

$R = \frac{|E_0''|^2}{|E_0|^2} = \frac{|X - iY|^2}{|X + iY|^2} = \frac{|Z^*|^2}{|Z|^2} = \frac{|Re^{-i\Theta}|^2}{|Re^{i\Theta}|^2} = 1$

"total internal reflection"



critical angle . $n'^2 - n^2 \sin^2 i \geq 0$

$r' = 90°$

$$\boxed{\sin i_c = \frac{n'}{n}}$$

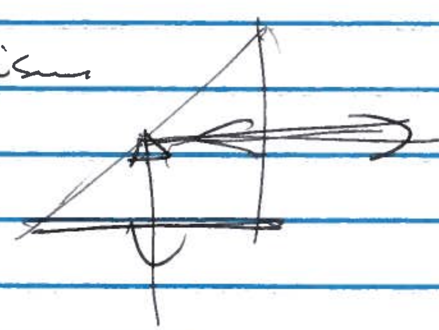
All of outside world ⟷ circle θ_c

beyond θ_c ⟶ silvery reflection

"crown glass" (K5) $n = 1.522$

dense flint (SF6) $n \simeq 1.805$

$n > \sqrt{2}$ ⟷ 45° prism



Binoculars

periscope . .

$\parallel$ $\quad \dfrac{E_0''}{E_0} = \dfrac{\frac{\mu}{\mu'}\, n'^2 \cos i - n\sqrt{n'^2 - n^2 \sin^2 i}}{\frac{\mu}{\mu'}\, n'^2 \cos i + n\sqrt{n'^2 - n^2 \sin^2 i}}$

$= 0$ $\quad \frac{\mu}{\mu'} \cdot n'^2 \cos i = n \sqrt{n'^2 - n^2 \sin^2 i}$

$$n'^4 \cos^2 i = n^2 n'^2 - n^4 \sin^2 i$$

$$\frac{n'^2 \cos^2 i}{n^2} + \frac{n^2 \sin^2 i}{n'^2} = 1$$

$$\cos^2 i = \frac{n^2}{n'^2 + n^2} \qquad \sin^2 i = \frac{n'^2}{n'^2 + n^2}$$

"Brewster's Angle". $\quad \boxed{\tan i_B = \frac{n'}{n}}$

unlike $\left(\sin i_c = \frac{n'}{n} \right)$, Always has a solution

Polarized reflected light

$\perp$. $E_0'' = 0 \rightarrow n \cos i = \sqrt{n'^2 - n^2 \sin^2 i}$

$$n^2 \cos^2 i + n^2 \sin^2 i = n^2 = n'^2$$

$\left. \begin{array}{l} n = 1 \\ n' = 1.3 \text{ (water)} \end{array} \right) \rightarrow \tan i_B = 1.3 \quad i_B = 52°$

Magnetohydrodynamics. MHD. $\boxed{\S\,7}$

given $\vec{E}, \vec{B}$ $\rightarrow$ $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) = \rho\vec{E} + \vec{J}\times\vec{B}$

given: $\rho, \vec{J}$ $\rightarrow$ $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$ $\quad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \cdots$

Each affects the other.

$\rho_m, \vec{v}, p$ Hydrodynamics

add $\vec{B}$ "MHD". add. $\rho, \frac{\partial\vec{E}}{\partial t}$ Plasma Physics

Hydrodynamics.

$$\boxed{\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0}$$

continuity
conservation of mass.

Euler $\quad \rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\vec{\nabla}p - \rho_m\vec{\nabla}\Phi_g - \rho_e\vec{\nabla}\Phi_e + \vec{J}\times\vec{B}$

$+ \cdots$

$$+ \frac{\partial}{\partial x_i}\left[\mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}(\vec{\nabla}\cdot\vec{v})\right]$$

Navier-Stokes

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}$$

"convective" derivative
(along flow)

Static background: $\rho = \rho_0$ $\quad \vec{v} = 0$

perturb. $\rho = \rho_0 + \delta\rho$ $\quad \vec{v} = \vec{0} + \delta\vec{v}$ $\quad \delta$ "small"

$$\frac{\partial}{\partial t}(\rho_0 + \delta\rho) + \vec{\nabla}\cdot\left[(\rho_0 + \cancel{\delta\rho})(\delta\vec{v})\right] = 0$$

$$\boxed{\frac{\partial}{\partial t}(\delta\rho) + \rho_0\, \vec{\nabla}\cdot(\delta\vec{v}) = 0}$$

$$(\rho_0 + \delta\rho)\left[\frac{\partial}{\partial t}(\delta\vec{v}) + \cancel{(\delta\vec{v}\cdot\vec{\nabla})\,\delta\vec{v}}\right) = -\vec{\nabla} p.$$

Equation of state: $p = p(\rho)$

$$p(\rho_0 + \delta\rho) = p(\rho_0) + \left(\frac{dp}{d\rho}\right)_0 \cdot \delta\rho + \cdots$$

$$\boxed{\rho_0 \frac{\partial}{\partial t}(\delta\vec{v}) = -\left(\frac{dp}{d\rho}\right)_0 \cdot \vec{\nabla}(\delta\rho)}$$

$$\frac{\partial^2}{\partial t^2}(\delta\rho) = -\rho_0 \underline{\frac{\partial}{\partial t}(\vec{\nabla}\cdot\delta\vec{v})}$$

$$\rho_0\, \underline{\vec{\nabla}\cdot\left(\frac{\partial}{\partial t}(\delta\vec{v})\right)} = -\left(\frac{dp}{d\rho}\right)_0 \cdot \nabla^2(\delta\rho).$$

$$\boxed{\nabla^2(\delta\rho) - \frac{1}{(dp/d\rho)_0}\frac{\partial^2}{\partial t^2}(\delta\rho) = 0}$$

$$c_s^2 = \left(\frac{dp}{d\rho}\right)_0 . \qquad \left(\nabla^2 - \frac{1}{c_s^2}\frac{\partial^2}{\partial t^2}\right)\delta\rho = 0.$$

Wave equation.

Ideal gas $\quad p = \rho \frac{kT}{m}.$

Isothermal $\quad \frac{dp}{d\rho} = \frac{kT}{m}.$

Adiabatic $\quad pV^\gamma = \text{constant}$

$p = \text{constant} \cdot \rho^\gamma \quad \Big| \quad \frac{dp}{d\rho} = \frac{\gamma p}{\rho} = \frac{\gamma kT}{m}$

N_2, O_2 . diatomic $\quad \gamma = \frac{7}{5}$

$$m = (0.8)(28) + (0.2)(32) = 28.8\, m_p.$$

$$kT = (1\,\text{eV})\left(\frac{T}{11{,}600\,K}\right) \simeq \frac{1}{40}\,\text{eV} \quad \left(\begin{array}{l}17C\\ = 62F\end{array}\right)$$

$$c_s^2 = \frac{(1.4)\left(\frac{1}{40}\text{eV}\right)}{(28.8)(0.938 \cdot 10^9\,\text{GeV}/c^2)} = 1.295 \times 10^{-12}\, c^2$$

$$c_s = 1.138 \times 10^{-6} \cdot c = 340\ \text{m/s}$$

$$= 1100\ \text{ft/s}$$