2|12|2017 MHD.

HD. $\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$

HD + B $\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\vec{\nabla}p + \vec{J}\times\vec{B}$

$+ \eta \frac{\partial}{\partial x_j}\left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}(\vec{\nabla}\cdot\vec{v})\right]$

unmaxwell $\frac{\partial \vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}\times\vec{B}) + \frac{1}{\mu\sigma}\nabla^2\vec{B}$

Perturb about ρ_0, $\vec{B_0}$, linearize

Conservative $\eta, \frac{1}{\sigma} \to 0$

$\frac{\partial}{\partial t}(\delta\rho) + \rho_0\vec{\nabla}\cdot(\delta\vec{v}) = 0$

$\rho_0\frac{\partial}{\partial t}(\delta\vec{v}) = -c_s^2\vec{\nabla}(\delta\rho) - \frac{1}{\mu}\vec{B_0}\times(\vec{\nabla}\times\delta\vec{B})$

$\frac{\partial}{\partial t}(\delta\vec{B}) = \vec{\nabla}\times(\delta\vec{v}\times\vec{B_0})$.

$v_A^2 = \frac{B_0^2}{\rho\mu}$

$\frac{\partial}{\partial t}(2) + \vec{\nabla}(1) + (3) \to$

$\frac{\partial^2}{\partial t^2}(\delta\vec{v}) = c_s^2\vec{\nabla}(\vec{\nabla}\cdot\delta\vec{v})$

$- \vec{v}_A\times(\vec{\nabla}\times[\vec{\nabla}\times(\delta\vec{v}\times\vec{v}_A)])$

7.72.

2

$$\delta \vec{v} = \vec{v}_1 \, e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

$$-\omega^2 \vec{v}_1 + c_s^2 \, \vec{k}(\vec{k}\cdot\vec{v}_1)$$

$$- \vec{v}_A \times [\vec{k} \times (\vec{k} \times (\vec{v}_1 \times \vec{v}_A))] = 0.$$

$$\vec{v}_1(\vec{k}\cdot\vec{v}_A) - \vec{v}_A(\vec{k}\cdot\vec{v}_1)$$

$$\to \; -(\vec{k}\cdot\vec{v}_A)[\vec{k}(\vec{v}_A\cdot\vec{v}_1) - \vec{v}_1(\underset{①}{\vec{k}\cdot\vec{v}_A})]$$

$$+ (\underset{②}{\vec{k}\cdot\vec{v}_1})[\vec{k}(\vec{v}_A\cdot\vec{v}_A) - \vec{v}_A(\underset{③}{\vec{k}\cdot\vec{v}_A})]$$

$$-\omega^2 \vec{v}_1 + (c_s^2 + v_A^2)\,\vec{k}(\vec{k}\cdot\vec{v}_1)$$

$$+ \underset{①}{(\vec{k}\cdot\vec{v}_A)^2 \vec{v}_1} - \underset{②}{\vec{k}(\vec{k}\cdot\vec{v}_A)(\vec{v}_A\cdot\vec{v}_1)}$$

$$- \underset{③}{\vec{v}_A(\vec{k}\cdot\vec{v}_A)(\vec{k}\cdot\vec{v}_1)} = 0$$

7.75

Eigenvalue : $\omega^2 \vec{v}_1 = M\vec{v}_1$

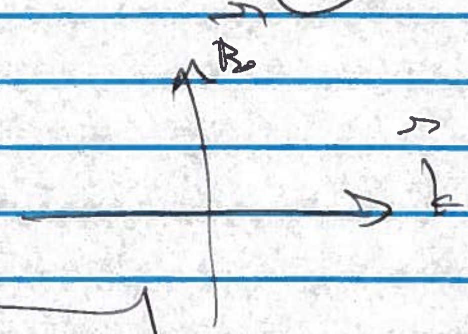
structure overall : $M\vec{v}_1 = \vec{0}$

$M^{-1} \to M^{-1}M\vec{v}_1 = \vec{v}_1 = M^{-1}\vec{0} = \vec{0}$

so: not invertible : $\det M = 0$ then

special cases

I. $\vec{k} \perp \vec{B}_0 \rightarrow \vec{k}\cdot\vec{v}_A = 0$.



$$-\omega^2 \vec{v}_1 + (c_s^2+v_A^2)\vec{k}(\vec{k}\cdot\vec{v}_1) = 0$$

in general: $\vec{v}_1 = \hat{k}(\hat{k}\cdot\vec{v}_1) + [\vec{v}_1 - \hat{k}(\hat{k}\cdot\vec{v}_1)] = \vec{v}_\parallel + \vec{v}_\perp$

$$-\omega^2(\vec{v}_\parallel+\vec{v}_\perp) + (c_s^2+v_A^2)k^2\vec{v}_\perp = 0$$

$\vec{v}_\perp = 0$ $\vec{v}_1 \parallel \vec{k}$

$-\omega^2 + (c_s^2+v_A^2)k^2 = 0$ $\omega^2 = (c_s^2+v_A^2)k^2$

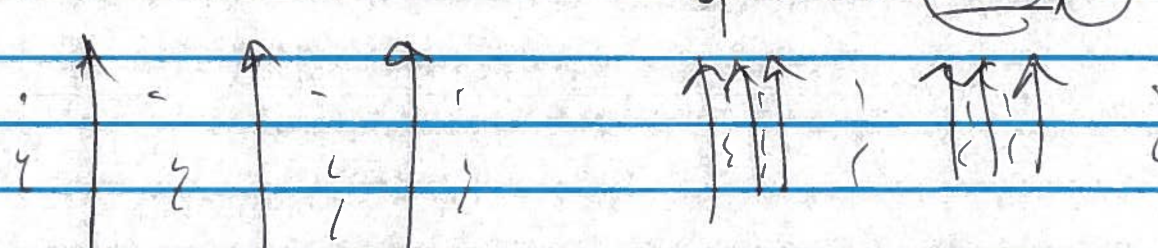
$v = \frac{\omega}{k} = \frac{d\omega}{dk} = \sqrt{c_s^2+v_A^2}$

$-i\omega\rho_1 + \rho_0(i\cancel{\vec{k}\cdot\vec{v}_1}) = 0$ (kv_1) $\rho_1 = \rho_0\frac{\hat{k}\cdot\vec{v}_1}{\omega} = \rho_0\frac{\hat{k}\cdot\vec{v}_1}{\sqrt{c_s^2+v_A^2}}$

$-i\omega\vec{B}_1 = i\vec{k}\times(\vec{v}_1\times\vec{B}_0) = i[\vec{v}_1(\cancel{\vec{k}\cdot\vec{B}_0}) - \vec{B}_0(\vec{k}\cdot\vec{v}_1)]$

$\vec{B}_1 = \vec{B}_0\frac{\hat{k}\cdot\vec{v}_1}{\sqrt{c_s^2+v_A^2}}$ $B_1 \parallel B_0$

squeezes matter $\vec{B}$



longitudinal magneto-sonic wave.

II. $\vec{k} \parallel \vec{B}_0\ (\vec{v}_A)$. $\quad \vec{k}(\vec{v}_A \cdot \vec{v}_1) = \vec{v}_A(\vec{k} \cdot \vec{v}_1)$

$$-\omega^2 \vec{v}_1 + \cancel{(c_s^2 + v_A^2)} \frac{k^2}{v_A^2} \vec{v}_A(\vec{v}_A \cdot \vec{v}_1)$$

$$+ k^2 v_A^2 \vec{v}_1 - k^2 \vec{v}_A(\vec{v}_A \cdot \vec{v}_1) - \vec{v}_A \cancel{k^2(\vec{v}_A \cdot \vec{v}_1)}$$

$$-\omega^2 \vec{v}_1 + k^2 v_A^2 \vec{v}_1 + \left(\frac{c_s^2}{v_A^2} - 1\right) k^2 \vec{v}_A(\vec{v}_A \cdot \vec{v}_1) = 0$$

II.A. $\vec{v}_A \cdot \vec{v}_1 = 0 \qquad \omega^2 = v_A^2 k^2 \qquad \vec{v}_1 \perp \vec{B}_0$

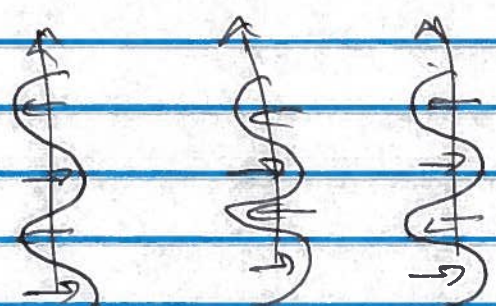
$$-i\omega \rho_1 + \rho_0 \cancel{(i \vec{k} \cdot \vec{v}_1)} = 0 \qquad \rho_1 = 0$$

$$-i\omega \vec{B}_1 = i\vec{k} \times (\vec{v}_1 \times \vec{B}_0) = i\left[\vec{v}_1(\vec{k} \cdot \vec{B}_0) - \vec{B}_0 \cancel{(\vec{k} \cdot \vec{v}_1)}\right]$$

$$\vec{B}_1 = -\vec{v}_1 \left(\frac{\vec{k} \cdot \vec{B}_0}{\omega}\right) = -\vec{v}_1 \frac{(\vec{k} \cdot \vec{B}_0)}{v_A k} = -\vec{v}_1 \frac{(\vec{v}_A \cdot \vec{B}_0)}{v_A} = -\vec{v}_1 B_0$$

$$\vec{B}_1 = -\frac{\vec{v}_1}{v_A} \vec{B}_0$$

Alfvén wave



no density fluctuation

"Pluck" $\vec{B}_0$

$\vec{B}_1 \perp \vec{B}_0$

$\vec{v}_1 \perp \vec{B}_0$

IIB. $\vec{k} \parallel \vec{B}_0$, $\vec{v}_1 \parallel \vec{B}_0$ also.

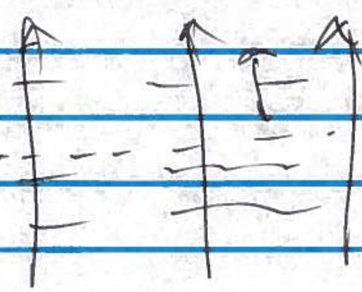
$$-\omega^2 + (c_s^2 + \cancel{v_A^2})k^2 + \cancel{k^2 v_A^2} - \cancel{k^2 v_A^2} - \cancel{k^2 v_A^2} = 0$$

$$\omega^2 = c_s^2 k^2$$

$$-i\omega\rho_1 + \rho_0\, i\vec{k}\cdot\vec{v}_1 = 0 \qquad \vec{v}_1 \parallel \vec{k}$$

$$\rho_1 = \frac{v_1}{c_s}\rho_0$$

$$-i\omega\vec{B}_1 = i\vec{k}\times\cancel{(\vec{v}_1\times\vec{B}_0)} = 0. \qquad \vec{B}_1 = 0$$



motion $\parallel \vec{B}$.
no magnetic force
sound wave