2/14/2017.

Skipping

Kramers Kronig relations

$$\boxed{\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \oint \frac{d\omega'}{2\pi i} \frac{(\varepsilon(\omega')/\varepsilon_0 - 1)}{\omega' - \omega}}$$

$$\left\{ \begin{array}{l} \mathrm{Re}\,\varepsilon = 1 + \oint \frac{d\omega'}{2\pi i} \frac{\mathrm{Im}(\varepsilon(\omega'))}{\omega - \omega'} \\ \\ \mathrm{Im}\,\varepsilon = - \oint \frac{d\omega'}{2\pi i} \frac{\mathrm{Re}(\varepsilon - 1)}{\omega - \omega'} \end{array} \right.$$

confined geometry (Ch. 8).

$k_x a = n\pi$

$$\int \frac{d\phi}{2\pi} e^{ik_z z} e^{ik\rho\cos\phi}$$

$$\to e^{ikz} J_0(k\rho).$$

$ka = x_{0n}$

2

Chapter 9 Radiating Systems

oscillating sources: $\rho, \vec{J} \sim e^{-i\omega t}$ Re[...]

Lorentz gauge: $\mu_0 \varepsilon_0 \frac{\partial \Phi}{\partial t} + \vec{\nabla} \cdot \vec{A} = 0$.

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Phi = -\rho/\varepsilon_0$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -\mu_0 \vec{J}$$

$\frac{\omega^2}{c^2}$ $\quad k = \frac{\omega}{c}$

$$\left(\nabla^2 + k^2\right)\Phi = -\rho/\varepsilon_0$$

$$\left(\nabla^2 + k^2\right)\vec{A} = -\mu_0 \vec{J}$$

"Helmholtz equation"

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi = \frac{\hbar^2 k^2}{2m}\psi$$

$$\longrightarrow \left(\nabla^2 + k^2\right)\psi = \frac{2mV}{\hbar^2}\psi$$

Green's function $G(\vec{x}, \vec{x}') = \dfrac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}$

$$(\nabla^2 + k^2) G = -4\pi\delta(\vec{x}-\vec{x}')$$ directly.

$$G(\vec{x}, t; \vec{x}', t') = \frac{1}{|\vec{x}-\vec{x}'|} \delta\left(t - t' - \tfrac{1}{c} R\right)$$

$$\vec{A}(\vec{x}, t) = \int d^3x' \, dt' \cdot \frac{\mu_0}{4\pi} \underbrace{\vec{J}(\vec{x}', t')}_{e^{-i\omega t'}} \frac{1}{|\vec{x}-\vec{x}'|} \delta\left(t - t' - \tfrac{1}{c} R\right)$$

$$= \int d^3x' \frac{\mu_0}{4\pi} \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|} e^{-i\omega t} e^{i\frac{\omega}{c}|\vec{x}-\vec{x}'|}$$

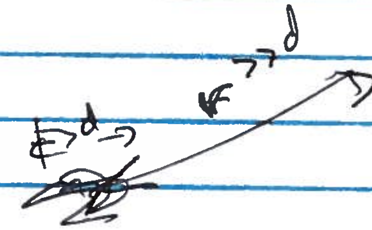
$$\boxed{\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int d^3x' \, \vec{J}(\vec{x}') \frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}} \; e^{-i\omega t}$$ (9.3)

- perturbation theory kd ≪ 1.
- exact solutions
- multipole expansion (vector $\vec{X}_{lm}$)

All start from localized sources

$$R = |\vec{x}-\vec{x}'| = \sqrt{r^2 + r'^2 - \cancel{2rr'\cos\gamma}}$$

$\vec{x}\cdot\vec{x}'$



$$= r\left(1 + \frac{r'^2}{r^2} - \frac{2\vec{x}\cdot\vec{x}'}{r^2}\right)^{1/2}$$

$$\simeq r\left(1 - \frac{\vec{x}\cdot\vec{x}'}{r^2}\right) = r - \hat{r}\cdot\vec{x}'$$

$$\boxed{R \simeq r - \hat{r}\cdot\vec{x}'} + O(1/r)$$

$$\boxed{\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3x'\, \vec{J}(\vec{x}')\, e^{-ik\hat{r}\cdot\vec{x}'}} \quad (9.8)$$

~~(xxx)~~

$$\vec{A}(\vec{x}) = \frac{e^{ikr}}{r}\vec{F}(\hat{r})$$ always in radiation region.

$$\vec{B} = \vec{\nabla}\times\vec{A} = \left(\hat{r}\frac{\partial}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial}{\partial\theta} + \cancel{\frac{\hat{\phi}}{r\sin\theta}\frac{\partial}{\partial\phi}}\right)\times\vec{A}$$

$$\frac{\partial}{\partial r}\left(\frac{e^{ikr}}{r}\right) = (ik)\left(\frac{e^{ikr}}{r}\right) - \cancel{\frac{e^{ikr}}{r^2}}$$

$$\boxed{\vec{B} = ik\hat{r}\times\vec{F}\,\frac{e^{ikr}}{r}}$$

5

$$\vec{\nabla}\times\vec{B} = ik\hat{r}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} = -i\frac{\omega}{c^2}\vec{E} = \frac{-ik}{c}\vec{E}$$

$$\vec{E} = -ick\,\hat{r}\times(\hat{r}\times\vec{F})\frac{e^{ikr}}{r}$$

$$\boxed{\vec{E} = +i\omega\,(\hat{r}\times\vec{F})\times\hat{r}\,\frac{e^{ikr}}{r}}$$

$$\vec{F}(\hat{r}\cdot\hat{r}) - \hat{r}(\vec{F}\cdot\hat{r}) = \vec{F}_\perp$$

Both $\vec{E}, \vec{B} \perp \hat{r}$, transverse

$$\vec{S} = \mathrm{Re}\left(\frac{1}{2\mu_0}\vec{E}\times\vec{B}^*\right)$$

$$= \frac{1}{2\mu_0}\mathrm{Re}\left[i\omega(\hat{r}\times\vec{F})\times\hat{r}\,\frac{e^{ikr}}{r}\times\left(ik\hat{r}\times\vec{F}\,\frac{e^{ikr}}{r}\right)^*\right]$$

$$= \frac{1}{2\mu_0}\frac{\omega k}{r^2}\left[\left[(\hat{r}\times\vec{F})\times\hat{r}\right]\times(\hat{r}\times\vec{F}^*)\right]$$

$$\hat{r}\,(\hat{r}\times\vec{F}\cdot\hat{r}\times\vec{F}^*)$$
$$-(\hat{r}\times\vec{F})\,\cancel{(\hat{r}\cdot(\hat{r}\times\vec{F}^*))}$$

$$\left\langle\frac{dP}{d\Omega}\right\rangle = r^2\,\hat{r}\cdot\langle\vec{S}\rangle = \frac{1}{2\mu_0}ck^2\,|\hat{r}\times\vec{F}|^2$$

$$\boxed{\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{ck^2}{2\mu_0}\left|(\hat{r}\times\vec{F})\times\hat{r}\right|^2}$$

$$\propto |\vec{E}|^2$$

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int d^3x' \, \vec{J}(\vec{x}') \, e^{-ik\hat{r}\cdot\vec{x}'}}$$

exact for $r \gg d$, radiation regime

$k\hat{r} = \vec{k}_{out}$

F.T. of $\vec{J}(\vec{x})$.

Approximation "long wavelength" $kd \ll 1$, $\omega d \ll c$

$\lambda \gg d$

$$e^{-ik\hat{r}\cdot\vec{x}} \simeq 1 - ik\hat{r}\cdot\vec{x} + \cdots$$

$\uparrow$ leading.

$$\vec{A}(\vec{x}) \simeq \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int d^3x' \, \vec{J}(\vec{x}')$$

static: $\int d^3x' \, \vec{J}(\vec{x}') = 0$, not any more

$$\nabla_j'(x_i' J_j') = \delta_{ij} \cdot J_j + x_i'(\nabla_j' J_j') = \vec{J} + \vec{x}'(\vec{\nabla}' \cdot \vec{J})$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = -i\omega\rho + \vec{\nabla}\cdot\vec{J} = 0 \qquad \boxed{\vec{\nabla}\cdot\vec{J} = i\omega\rho}$$

$$\int d^3x' \, \vec{J}(\vec{x}') = \int d^3x' \left[\cancel{\nabla_j'(x_i' J_j)} - \vec{x}'(\vec{\nabla}' \cdot \vec{J}) \right]$$

$$= \int d^3x' (-i\omega\rho) \vec{x}' = -i\omega \vec{p}$$

Electric dipole moment = constant

$$\boxed{\vec{A}_{E.D.} = -i\omega \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \vec{p}} \qquad (9.16).$$

charge conserved $\rightarrow$ can't have oscillating Q"

[GR : mass, momentum conserved.
center of mass fixed $\rightarrow$ leading Quadrupole]

$$\vec{H} = \frac{1}{\mu_0} \vec{\nabla}\times\vec{A} = \frac{1}{\cancel{\mu_0}}\left(\hat{r}\frac{d}{dr}\right)\times\left(-i\omega \frac{\cancel{\mu_0}}{4\pi} \frac{e^{ikr}}{r}\vec{p}\right)$$

$$= \frac{-i\omega}{4\pi}\left(ik - \frac{1}{r}\right)\hat{r}\times\vec{p}\,\frac{e^{ikr}}{r}.$$

$$\boxed{\vec{H}_{E.D.} = \frac{ck^2}{4\pi}\hat{r}\times\vec{p}\,\frac{e^{ikr}}{r}\left(1 - \frac{1}{ikr}\right)}$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t} = -i\omega\varepsilon_0\vec{E} = -ick\varepsilon_0\vec{E} \qquad 9.20$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[k^2(\hat{r}\times\vec{p})\times\hat{r}\,\frac{e^{ikr}}{r} + \frac{\left[3\hat{r}(\hat{r}\cdot\vec{p}) - \vec{p}\right]}{r^3}e^{ikr}(1 - ikr)\right]}$$