2/28/2018

dipole

electric dipole

$$P = \frac{c^2 Z_0 k^4}{12\pi} |\vec{p}|^2 = \frac{1}{3} c k^4 \frac{|\vec{p}|^2}{4\pi\varepsilon_0} \quad (9.24)$$

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0 k^4}{32\pi^2} |\hat{r} \times \vec{p}|^2$$

$\rightarrow \sin^2\theta$

$\downarrow$

magnetic dipole

$$\boxed{P = \frac{c^2 Z_0 k^4}{12\pi} \left|\frac{\vec{m}}{c}\right|^2} \quad (9.24)$$

electric quadrupole

$$\boxed{P = \frac{c^2 Z_0 k^6}{1440\pi} |Q_{ij}|^2} \quad (9.49)$$

$$P = \frac{c^2 Z_0 k^6}{1152\pi^2} \cdot \frac{4\pi}{15} \cdot Q_{ij} Q_{ij}^*$$

$$\boxed{P = \frac{c^2 Z_0 k^6}{1440\pi} |Q_{ij}|^2} \qquad (9.49)$$

Gravity

$$c Z_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{\varepsilon_0}$$

$$P = \frac{1}{360} \cdot c k^6 \cdot \frac{|Q_{ij}|^2}{4\pi\varepsilon_0}$$

Analogy. $\frac{q^2}{4\pi\varepsilon_0} \to G m^2$

$$Q_{ij} = \int d^3x \, (3 x_i x_j - \delta_{ij} r^2)$$

$$I_{ij} = \int d^3x \left(x_i x_j - \frac{1}{3} \delta_{ij} r^2 \right)$$

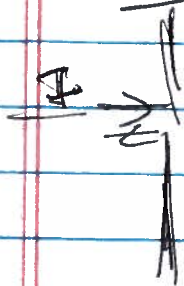
$$P \to \frac{1}{360} \cdot c k^6 G (9 I_{ij})^2 = \frac{1}{40} c k^6 G |I_{ij}|^2$$

GR

$$\boxed{P = \frac{1}{40} \cdot c k^6 \cdot G |I_{ij}|^2}$$

Rotating

"Exact" toy model. Linear antenna §9.4



center-fed. $I = I(|z|)$. symmetric.

$$\int d^3x' \, \vec{J}(x') \longrightarrow \int dz' \, \hat{z} \, I(z')$$

Near antenna. $\vec{I} \propto \hat{z} \longrightarrow \vec{A} \propto \hat{z}$

just outside. $(\nabla^2 + k^2)\vec{A} = 0 \rightarrow \left(\frac{\partial^2}{\partial z^2} + k^2\right) A = 0$ $\quad A \sim e^{\pm ikz}$, $\quad I \sim e^{\pm ikz}$

$$\boxed{I(z) = I_0 \sin\left[k\left(\frac{d}{2} - |z|\right)\right]}$$

$a \ll d$.

losses (radiation, ohmic) $\cancel{e^{-kz}}$

Charge. $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$

$$\boxed{\frac{\partial \lambda}{\partial t} + \frac{\partial I}{\partial z} = 0} \qquad i\omega\lambda = \frac{\partial I}{\partial z}$$

$$i\omega\lambda = \mp k I_0 \cdot \cos\left[k\left(\frac{d}{2} - |z|\right)\right]$$

$$\boxed{\lambda = \pm i \frac{I_0}{c} \cdot \cos\left[k\left(\frac{d}{2} - |z|\right)\right]}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int_{-d/2}^{d/2} dz' \cdot I_0 \hat{z} \cdot \sin\left[k\left(\frac{d}{2} - |z'|\right)\right] e^{-ikz'\cos\theta}$$

$(z' < 0)$ $\quad (z'' = -z')$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r} I_0 \hat{z} \int_0^{d/2} dz' \cdot \sin\left[k\left(\frac{d}{2} - z'\right)\right] \left(e^{-ikz'\cos\theta} + e^{+ikz'\cos\theta}\right) = 2\cos(kz'\cos\theta)$$

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b.$$

$$2\sin a \cos b = \sin(a+b) + \sin(a-b)$$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{kr} 2 I_0 \hat{z} \left[\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{1 - \cos^2\theta}\right]$$

(9.55)

$$\vec{H} = ik\hat{r} \times \vec{A} \quad \leftarrow \sin\theta$$

$$\vec{E} = Z_0 \vec{H} \times \hat{r}$$

$$\frac{dP}{d\Omega} = r^2 \hat{r} \cdot \vec{S}$$

$$\frac{dP}{d\Omega} = \frac{Z_0 |I_0|^2}{8\pi^2} \frac{\left[\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)\right]^2}{\sin^2\theta}$$

(9.56)

(B)

poles: $\sin^2\theta \to 0$, But, near poles. $\theta = \Delta\theta$, $\theta = \pi - \Delta\theta$ $\Big\}$ $\cos\theta = 1 - \frac{1}{2}\Delta\theta^2$

$$\cos\left(\frac{kd}{2}\cos\theta\right) \simeq \cos\left(\frac{kd}{2} - \frac{1}{2}\left(\Delta\theta\right)^2 \frac{kd}{2}\right)$$

$$= \cos\frac{kd}{2} \cdot \cancel{\cos\left(\Delta\theta^2\right)} + \sin\frac{kd}{2} \cdot \frac{1}{2}\left(\frac{kd}{2}\right)\Delta\theta^2$$

$$\frac{dP}{d\Omega} \to \frac{Z_0 I_0^2}{8\pi} \cdot \frac{\left[\frac{1}{4} kd \sin\frac{kd}{2} \cdot \cancel{\Delta\theta^2}\right]^2}{\cancel{(\Delta\theta)^2}} \sim \underline{(\Delta\theta)^2}$$

$\frac{dP}{d\Omega} \to 0$ at poles.

equator: $\theta = \frac{\pi}{2}$, $\cos\theta \to 0$ $\left(1 - \cos\frac{kd}{2}\right)^2$

max $\cos\frac{kd}{2} \simeq -1$ $\frac{kd}{2} = \pi, 3\pi \ldots$ $\left(kd = 2\pi(2n+1)\right)$

zero $\cos\frac{kd}{2} = 1$. $\frac{kd}{2} = 2\pi n$ $kd = 4\pi \cdot n$

$2\pi \cdot 2n$

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Case: $kd=\pi$, "half-wave" antenna.

$$\cos\frac{kd}{2} = \cos\frac{\pi}{2} = 0$$

$$\boxed{\frac{dP}{d\Omega} = \frac{Z_0|I_0|^2}{8\pi}\,\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}}$$

Case $kd=2\pi$ "full-wave antenna"

$$\cos\frac{kd}{2} = \cos\pi = -1$$

$$\boxed{\frac{dP}{d\Omega} = \frac{Z_0|I_0|^2}{8\pi}\,\frac{[1+\cos(\pi\cos\theta)]}{\sin^2\theta}}$$

$$\cos^2 x = \tfrac{1}{2}(1+\cos 2x).$$

$$\boxed{\frac{dP}{d\Omega} = \frac{Z_0|I_0|^2}{8\pi^2}\,4\,\frac{\cos^4(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}}$$

$I = I(|z|)$