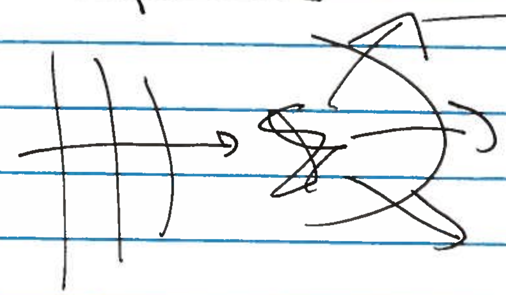
3/14/2018 Chapter 10: scattering and diffraction



radiation from source driven by incoming.

cross section = spherical transmission / reflection

$$\frac{d\sigma}{d\Omega} = \frac{(\text{out})}{(\text{in})} = \frac{(\text{out per solid angle})}{(\text{in per area})} = \frac{(dP/d\Omega)_{out}}{(dP/da)_{in}}.$$

In: $\vec{E}_{in} = \vec{E}_0 \, e^{i(\vec{k}\cdot\vec{x} - \omega t)}$

out: $\vec{E}_{out} = \vec{E}_{sc} = \frac{e^{ikr}}{r} \vec{F}(\theta,\phi) \, e^{-i\omega t}.$

$$\langle \vec{S}_{in} \rangle = \frac{1}{2} \vec{E} \times \vec{H}^* = \frac{1}{2Z_0} \vec{E}_0 \times (\hat{k} \times \vec{E}_0) = \frac{\hat{k}}{2Z_0} |\vec{E}_0|^2$$

$$\langle \vec{S}_{sc} \rangle = \frac{\hat{r}}{2Z_0} \frac{|\vec{F}|^2}{r^2}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{r^2 \, \hat{r}\cdot\vec{S}_{sc}}{\hat{k}\cdot\vec{S}_{in}} = \frac{|\vec{F}|^2}{|\vec{E}_0|^2}}$$

Polarization $\rightarrow$ $|\vec{\varepsilon}^{\,*}\cdot\vec{F}|^2$ $\quad$ $|\vec{\varepsilon}_0\cdot\vec{E}_0|^2$

often summed over $\vec{\varepsilon}$
averaged over $\vec{\varepsilon}_0$

Example: "small" dielectric sphere. $\mu = \mu_0$
$(ka \ll 1)$. $\varepsilon = K \cdot \varepsilon_0$.

$$\vec{E}_{inside} = \frac{3}{K+2} \vec{E}_0 \left(e^{-i\omega t}\right).$$

Ch 4. $\langle \vec{E} \rangle = -\frac{1}{3}\frac{\vec{P}}{\varepsilon_0} + \vec{E}_0$

$$\vec{P} = \varepsilon_0 \chi \vec{E}_{in} = \varepsilon_0 (K-1) \left(\frac{3\vec{E}_0}{K+2}\right).$$

$$\vec{p} = \frac{4\pi}{3} a^3 \cdot \vec{P} = 4\pi\varepsilon_0 \cdot \left(\frac{K-1}{K+2}\right) \vec{E}_0 a^3.$$

$$\left(\frac{dP}{da}\right)_{in} = \frac{|\vec{E}_0|^2}{2Z_0}.$$

$$\left(\frac{dP}{d\Omega}\right)_{out} = \frac{c^2 Z_0 k^4}{32\pi^2} \left|(\hat{r} \times \vec{p}) \times \hat{r}\right|^2 \qquad (9.22)$$

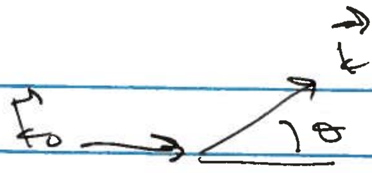
$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\frac{c^2 Z_0 k^4}{32\pi^2} \left|\frac{K-1}{K+2}\right|^2 a^6 \cdot (4\pi\varepsilon_0)^2 \left|\hat{r} \times \vec{E}_0\right|^2}{\frac{1}{2} Z_0 |\vec{E}_0|^2}$$

$$= \frac{c^2 Z_0^2 \varepsilon_0^2 \cdot 2(4\pi)^2 \cdot k^4 a^6}{32\pi^2} \left|\frac{K-1}{K+2}\right|^2 \left|(\hat{r} \times \vec{\varepsilon}_0) \times \hat{r}\right|^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = \underbrace{k^4 a^6}_{\text{(size)}} \cdot \underbrace{\left|\frac{K-1}{K+2}\right|}_{\text{(material)}} \cdot \underbrace{\left|(\hat{r} \times \vec{\varepsilon}_0) \times \hat{r}\right|^2}_{\text{(orientation)}}}$$

incident polarization

Specific geometry.



$$\hat{k}_0 = \hat{z}$$

$$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta .$$

$$\hat{\varepsilon}_0 = \hat{x}\cos\phi_0 + \hat{y}\sin\phi_0 .$$

$$\hat{r}\times\hat{\varepsilon}_0 = -\cos\theta\cdot\sin\phi_0\,\hat{x} + \cos\theta\cdot\cos\phi_0\,\hat{y} + \sin\theta\left(\cos\phi\cdot\sin\phi_0 - \sin\phi\cos\phi_0\right)\hat{z}$$

$$\hat{r}\times\vec{\varepsilon}_0 = \cos\theta\cdot\hat{\phi}_0 + \sin\theta\cdot\sin(\phi-\phi_0)\cdot\hat{z}$$

$$\left|\hat{r}\times\vec{\varepsilon}_0\right|^2 = \cos^2\theta + \sin^2\theta\cdot\sin^2(\phi-\phi_0)$$

Unpolarized incident $\Longleftrightarrow$ average over ϕ_0.

$$\langle\sin^2(\phi-\phi_0)\rangle = \tfrac{1}{2}.$$

$$\cos^2\theta + \tfrac{1}{2}\sin^2\theta = \tfrac{1}{2}\left(1+\cos^2\theta\right)$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left|\frac{\kappa-1}{\kappa+2}\right|^2 k^4 a^6\cdot\frac{1}{2}\left(1+\cos^2\theta\right)}$$

$$\boxed{\sigma = \frac{8\pi}{3}\left|\frac{\kappa-1}{\kappa+2}\right|^2\cdot k^4 a^6}$$

$$\sigma \propto k^4 \propto \frac{1}{\lambda^4}$$

$$\frac{\text{blue}}{\text{red}} = \frac{(700)^4}{(400)^4} = \frac{2401}{256} = 9.3789$$

Polarized detector choose ($\phi = 0$):

$\hat{k}_0 = \hat{z}$ $\hat{r} = \hat{x}\sin\theta + \hat{z}\cos\theta$

outgoing polarization $\vec{\varepsilon}_\perp = \hat{y}$ $\varepsilon_\parallel = \hat{z}\sin\theta - \hat{x}\cos\theta$

$$\vec{E}_{detected} \propto \vec{\varepsilon}^{\,*} \cdot \left[(\hat{r} \times \vec{\varepsilon}_0) \times \hat{r} \right]$$

$$= \vec{\varepsilon}^{\,*} \cdot \left[\vec{\varepsilon}_0 (\hat{r} \cdot \hat{r}) - \hat{r}(\hat{r} \cdot \vec{\varepsilon}_0) \right]$$

$$= \vec{\varepsilon}^{\,*} \cdot \vec{\varepsilon}_0 \quad (\hat{r} \cdot \hat{r}) - (\hat{r} \cdot \vec{\varepsilon}_0)\cancel{(\hat{r} \cdot \vec{\varepsilon}^{\,*})} = \vec{\varepsilon}^{\,*} \cdot \vec{\varepsilon}_0$$

$$\left(\frac{d\sigma_\perp}{d\Omega}\right) = \left|\frac{\kappa-1}{\kappa+2}\right|^2 k^4 a^6 \left| \hat{y} \cdot (\hat{x}\cos\phi_0 + \hat{y}\sin\phi_0) \right|^2$$

$$\boxed{\frac{d\sigma_\perp}{d\Omega} = \left|\frac{\kappa-1}{\kappa+2}\right|^2 k^4 a^6 \cdot \sin^2\phi_0}$$

$$\left(\frac{d\sigma_\parallel}{d\Omega}\right) = \cdots \left| (\hat{z}\sin\theta - \hat{x}\cos\theta) \cdot (\hat{x}\cos\phi_0 + \hat{y}\sin\phi_0) \right|^2$$

$$\boxed{\left(\frac{d\sigma_\parallel}{d\Omega}\right) = \left|\frac{\kappa-1}{\kappa+2}\right|^2 k^4 a^6 \cdot \cos^2\theta \cos^2\phi_0}$$

unpolarized $\langle \cos^2\phi_0 \rangle = \langle \sin^2\phi_0 \rangle = \frac{1}{2} \longrightarrow \cdots$