3|12|2018 $\frac{d\sigma}{d\Omega} = \frac{(dP/d\Omega)_{out}}{(dP/da)_{in}}$

dielectric sphere. $\vec{E}_{in} = \frac{3}{K+2}\vec{E}_0$.

$$\frac{d\sigma}{d\Omega} = \left|\frac{K-1}{K+2}\right|^2 \cdot k^4 a^6 \cdot \left|(\hat{r}\times\vec{\varepsilon}_0)\times\hat{r}\right|^2$$

Polarized detector. sees $(\vec{\varepsilon}^* \cdot \vec{E})$

$\hat{k}_0 = \hat{z}$ $\quad \hat{r} = \hat{x}\sin\theta + \hat{z}\cos\theta$ choose origin so that $\phi = 0$.

let $\vec{\varepsilon}_\perp = \hat{y}$ $\quad \vec{\varepsilon}_\parallel = \hat{x}\cos\theta - \hat{z}\sin\theta$ (I changed this).

$$\vec{\varepsilon}^* \cdot \left[(\hat{r}\times\vec{\varepsilon}_0)\times\hat{r}\right] = \vec{\varepsilon}^* \cdot \left[\vec{\varepsilon}_0(\hat{r}\cdot\hat{r}) - \hat{r}(\hat{r}\cdot\vec{\varepsilon}_0)\right]$$

$$= \vec{\varepsilon}^* \cdot \vec{\varepsilon}_0$$

|| Special case ($N = mg\cos\theta$;) $\to 0$.

②

$$\frac{d\sigma_\perp}{d\Omega} = \left|\frac{k-1}{k+2}\right|^2 k^4 a^6 \left|\hat{y}\cdot\left(\hat{x}\cos\phi_0 + \hat{y}\sin\phi_0\right)\right|^2$$

$$\boxed{\frac{d\sigma_\perp}{d\Omega} = \left|\frac{k-1}{k+2}\right|^2 k^4 a^6 \cdot \sin^2(\phi-\phi_0)}$$

$$\frac{d\sigma_{\parallel}}{d\Omega} = \left|\frac{k-1}{k+2}\right|^2 \cdot k^4 a^6 \left|(\hat{x}\cos\theta - \hat{z}\sin\theta)\cdot(\hat{x}\cos\phi_0 + \hat{y}\sin\phi_0)\right|^2$$

$$\boxed{\frac{d\sigma_{\parallel}}{d\Omega} = \left|\frac{k-1}{k+2}\right|^2 \cdot k^4 a^6 \cos^2\theta \cdot \cos^2(\phi-\phi_0)}$$

$\perp \to 0$ at $\phi=\phi_0$. aligned.

$\parallel \to 0$ at $\phi \perp \phi_0$. crossed.

Summed outgoing $\to$ $\left(\sin^2(\phi-\phi_0) + \cos^2(\phi-\phi_0)\cos^2\theta\right)$

before. $\boxed{\cos^2\theta + \sin^2\theta\cdot\sin^2(\phi-\phi_0)}$

$$\underbrace{\cos^2\theta\left(\sin^2\phi + \cos^2\phi\right) + \sin^2\theta}\sin^2\phi$$

$$= \sin^2\phi + \cos^2\phi\cos^2\theta$$

Averaged over ϕ_0 $\to$ $\frac{1}{2}\left(1+\cos^2\theta\right)$

K_e, K_m:

$$\vec{E}_{in} = \frac{3}{K_e+2}\,\vec{E}_0 \qquad \vec{B}_{in} = \frac{3K_m}{K_m+2}\,\vec{B}_0$$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} = 3\varepsilon_0\left(\frac{K_e-1}{K_e+2}\right)\vec{E}_0$$

$K_e \to \infty$

conductivity

$$\vec{M} = \chi_m \vec{H} = \frac{3}{\mu_0}\left(\frac{K_m-1}{K_m+2}\right)\vec{B}_0$$

$K_m \to 0$

$$\frac{1}{4\pi\varepsilon_0}\vec{p} = \vec{E}_0 a^3 \qquad \frac{\mu_0}{4\pi}\vec{m} = -\frac{1}{2}\vec{B}_0 a^3$$

$$\frac{m}{p} = \frac{-\frac{1}{2}\frac{4\pi}{\mu_0}B_0 a^3}{4\pi\varepsilon_0 \cdot E_0 a^3} = -\frac{1}{2}\cdot c^2 \cdot \frac{B_0}{E_0} = -\frac{1}{2}c$$

$$\frac{m/c}{p} = -\frac{1}{2}$$

Both of equal importance (even for $ka \ll 1$)

$$\vec{E}_{sc} = \frac{k^2}{4\pi\varepsilon_0}\,\frac{e^{ikr}}{r}\left[(\hat{r}\times\vec{p})\times\hat{r} - \hat{r}\times\left(\frac{\vec{m}}{c}\right)\right]$$

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$$\vec{\varepsilon}^{\,*}\cdot\vec{E}_{sc} \Rightarrow \vec{\varepsilon}^{\,*}\cdot\left[(\hat{r}\times\vec{p})\times\hat{r} \quad - \hat{r}\times\frac{\vec{m}}{c}\right)$$

$$\vec{\varepsilon}^{\,*}\cdot\left[(\hat{r}\times\vec{\varepsilon}_0)\times\hat{r}\right] = \vec{\varepsilon}^{\,*}\cdot\left[\,\vec{\varepsilon}_0(\hat{r}\cdot\hat{r}) - \cancel{\hat{r}(\hat{r}\cdot\vec{\varepsilon}_0)}\right]$$

(as before) $= \vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0$

$\vec{B}_0 \sim \hat{k}_0\times\varepsilon_0$

$$\vec{\varepsilon}^{\,*}\cdot\left[\hat{r}\times(\hat{k}_0\times\vec{\varepsilon}_0)\right] = (\vec{\varepsilon}^{\,*}\times\hat{r})\cdot(\hat{k}_0\times\vec{\varepsilon}_0)$$

$$= -(\hat{r}\times\vec{\varepsilon}^{\,*})\cdot(\hat{k}_0\times\vec{\varepsilon}_0).$$

$$\frac{d\sigma}{d\Omega} = k^4 a^6 \left|\underbrace{(\vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0)}_{\text{electric}} - \underbrace{\tfrac{1}{2}(\hat{r}\times\vec{\varepsilon}^{\,*})\cdot(\hat{k}_0\times\vec{\varepsilon}_0)}_{\text{magnetic}}\right|^2 \qquad (10.14)$$

2nd term: $(-1)^3$:

$\vec{m} = -\frac{1}{2}\vec{B}_0 a^3$

$\vec{E} \propto -\hat{r}\times\vec{m}$

$\vec{\varepsilon}^{\,*}\cdot\ldots \rightarrow -(\hat{r}\times\vec{\varepsilon}^{\,*})\cdot\ldots$

Geometry'

$\hat{k}_0 = \hat{z}$ $\qquad \hat{r} = \hat{z}\cos\theta + \hat{x}\sin\theta$

$\vec{\varepsilon}_\perp = \hat{y}$ $\qquad \vec{\varepsilon}_\parallel = \hat{x}\cos\theta - \hat{z}\sin\theta$

(1). $\vec{\varepsilon}_\parallel \cdot \vec{\varepsilon}_0 = \cos\phi_0 \cdot \cos\theta$

$$(\hat{r}\times\vec{\varepsilon}_\parallel)\cdot(\hat{k}_0\times\vec{\varepsilon}_0) = \underbrace{(\hat{y})}_{(\vec{\varepsilon}_\perp)}\cdot(-\sin\phi_0\hat{x} + \cos\phi_0\hat{y}) = \cos\phi_0.$$

(2) $\vec{\varepsilon}_\perp \cdot \vec{\varepsilon}_0 = \sin\phi_0.$

$$(\hat{r}\times\vec{\varepsilon}_\perp)\cdot(\hat{k}_0\times\vec{\varepsilon}_0) = \underbrace{(-\cos\theta\hat{x} + \sin\theta\hat{z})}_{(-\vec{\varepsilon}_\parallel)}\cdot(-\sin\phi_0\hat{x} + \cos\phi_0\hat{y})$$

$$= \sin\phi_0 \cdot \cos\theta.$$

$$\frac{d\sigma_\parallel}{d\Omega} = k^4 a^6 \cdot \left(\cos\theta - \tfrac{1}{2}\right)^2 \cdot \cos^2(\phi - \phi_0)$$

$$\frac{d\sigma_\perp}{d\Omega} = k^4 a^6 \cdot \left(1 - \tfrac{1}{2}\cos\theta\right)^2 \cdot \sin^2(\phi - \phi_0)$$

Unpolarized: $\langle \sin^2(\phi-\phi_0)\rangle = \langle \cos^2(\phi-\phi_0)\rangle = \frac{1}{2}$.

$(\parallel) = \frac{1}{2} k^4 a^6 \cdot \left(\cos\theta - \frac{1}{2}\right)^2$

$(\perp) = \frac{1}{2} k^4 a^6 \left(1 - \frac{1}{2}\cos\theta\right)^2$

Total $\frac{d\sigma}{d\Omega} = \frac{1}{2} (ka)^4 \cdot a^2 \left[\frac{5}{4}(1+\cos^2\theta) - 2\cos\theta\right]$

(1 from electric, $\frac{1}{4}$ from magnetic) interference

$\sigma = \int \sin\theta \, d\theta \, d\phi \rightarrow \frac{5}{4} \cdot k^4 a^6 \cdot \frac{8\pi}{3}$ | $\sigma = \frac{10\pi}{3} k^4 a^6$

Asymmetry Forward $\frac{5}{4} \cdot 2 - 2 = \frac{1}{2}$

Backwards $\frac{5}{4} \cdot 2 + 2 = \frac{9}{2}$

$\int_0^1 d\mu \left[\frac{5}{8}(1+\mu^2) - \mu\right] = \frac{5}{8}\left(1+\frac{1}{3}\right) - \frac{1}{2} = \frac{1}{3}$

$\int_{-1}^0 d\mu \left[\frac{5}{8}(1+\mu^2) - \mu\right] = \frac{5}{8}\left(1+\frac{1}{3}\right) + \frac{1}{2} = \frac{4}{3}$

$\Pi = \frac{(\perp) - (\parallel)}{(\perp) + (\parallel)} = \frac{\left(1-\frac{1}{2}\cos\theta\right)^2 - \left(\cos\theta - \frac{1}{2}\right)^2}{\left(1-\frac{1}{2}\cos\theta\right)^2 + \left(\cos\theta - \frac{1}{2}\right)^2}$

$= \frac{3\sin^2\theta}{5(1+\cos^2\theta) - 8\cos\theta}$

$\cos\theta = \frac{1}{2}$