3/29/2018

$$\frac{d\sigma_{\parallel}}{d\Omega} = k^4 a^6 \cdot \left(\cos\theta - \tfrac{1}{2}\right)^2 \cdot \cos^2(\phi-\phi_0)$$

$$\frac{d\sigma_{\perp}}{d\Omega} = k^4 a^6 \left(1 - \tfrac{1}{2}\cos\theta\right)^2 \cdot \sin^2(\phi-\phi_0).$$

total unpolarized $= k^4 a^6 \ \frac{1}{2}\left(\frac{5}{4}(1+\cos^2\theta) - 2\cos\theta\right)$

$$\Pi = \frac{(\perp)-(\parallel)}{(\perp)+(\parallel)}.$$

$$\sigma = \int \sin\theta \, d\theta \, d\phi = k^4 a^6 \cdot \frac{5}{4} \cdot \frac{8\pi}{3} = \frac{10\pi}{3} k^4 a^6.$$

ka not small $\longrightarrow$ multipole expansion

(3)

Recall. $\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} - \frac{i}{r}\hat{r}\times\vec{L}$

$$\vec{\nabla}\times\left(j_l \vec{X}\right) = i\hat{r}\frac{\sqrt{l(l+1)}}{r} j_l Y_{lm} + \frac{1}{r}\frac{\partial}{\partial r}(r j_l)\cdot \hat{r}\times\vec{X}_{lm}$$

$$\int d\Omega\, \vec{X}_{lm}^{*}\cdot\vec{E}_0 = \int d\Omega \left(\frac{\vec{L}Y_{lm}}{\sqrt{l(l+1)}}\right)^{*}\cdot(\hat{x}\pm i\hat{y})\, e^{ikz}$$

$$= \int d\Omega \left[\frac{(\hat{x}\mp i\hat{y})\vec{L}Y_{lm}}{\sqrt{l(l+1)}}\right] e^{ikz}.$$

Showed: $e^{i\vec{k}\cdot\vec{r}} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} 4\pi i^l j_l(kr)\, Y_{lm}^{*}(\hat{k})Y_{lm}(\hat{r})$

(10.43)

$\hat{k}=\hat{z}$ $\quad \theta=0$ $\quad m=0$. $\quad \sqrt{\frac{2l+1}{4\pi}}P_l(0)$

$$\boxed{e^{ikz} = \sum_{l'=0}^{\infty} i^{l'}\sqrt{4\pi(2l'+1)}\, j_{l'}(kr)\, Y_{l'0}(\theta,\phi)}$$

(10.43).

$$j_l(kr)\, a_{lm}^{\pm} = \sum_{l'=0}^{\infty}\int d\Omega \left(\frac{L_{\mp}Y_{lm}}{\sqrt{l(l+1)}}\right)^{*} \sqrt{4\pi(2l'+1)}\, i^{l'} j_{l'}(kr)\, Y_{l',0}(\theta,\phi)$$

$l'=l$

$$a_{lm}^{\pm} = i^l\frac{\sqrt{4\pi(2l+1)}}{\sqrt{l(l+1)}}\int d\Omega \left(L_{\mp}Y_{lm}\right)^{*} Y_{l,0}$$

Circularly polarized incident.

$$\vec{E}_0 = \vec{\varepsilon}_\pm E_0 \, e^{i\vec{k}_0\cdot\vec{x}} = (\hat{x} \pm i\hat{y})\, e^{ikz} \qquad \underline{\underline{|E_0|^2 = 2}}$$

$$\vec{\nabla}\times\vec{E} = i\vec{k}\times\vec{E} = i\vec{k}_0\times\vec{\varepsilon}_\pm E_0 \, e^{ikz}.$$

$$= ik\hat{z}\times(\hat{x}\pm i\hat{y})e^{ikz} = \pm k.(\hat{x}\pm i\hat{y})e^{ikz}$$

$$= -\frac{\partial\vec{B}}{\partial t} = +i\omega\vec{B} = ick\vec{B} \qquad \boxed{c\vec{B}_0 = \mp i\vec{E}_0}$$

multiple: write.

$$\vec{E}_0 = \sum_{l,m}\left[a^\pm_{lm} j_l(kr)\vec{X}_{lm} + \frac{i}{k} b^\pm_{lm}\vec{\nabla}\times(j_l\vec{X}_{lm})\right]$$

$$c\vec{B}_0 = \sum_{l,m}\left[\frac{-i}{k}a^\pm_{lm}\vec{\nabla}\times(j_l\vec{X}_{lm}) + b^\pm_{lm}\vec{\nabla}\times(j_l\vec{X}_{lm})\right]$$

orthogonal $\rightarrow$ $\displaystyle\int d\Omega\, \vec{X}^*_{lm}\cdot\vec{E}_0 = a^\pm_{lm} j_l(kr)$

$$\int d\Omega\, \vec{X}^*_{lm}\cdot c\vec{B}_0 = b^\pm_{lm} j_l(kr)$$

(4)

$m = \pm 1$

$$L_- Y_{l1} = \sqrt{(l+m)(l-m+1)}\, Y_{l,0} \overset{m=1}{=} \sqrt{l(l+1)}\, Y_{l,0}$$

$$L_+ Y_{l,-1} = \sqrt{(l-m)(l+m+1)}\, Y_{l,0} = \sqrt{l(l+1)}\, Y_{l,0}.$$

$m=-1$

$$\boxed{a^{\pm}_{lm} = i^l \sqrt{4\pi(2l+1)}\, \delta_{m,\pm 1}}$$

$$\boxed{b^{\pm}_{lm} = \mp i\, a^{\pm}_{lm}}$$

(10.53)

$$\vec{E}_0 = \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left[j_l(kr)\, \vec{X}_{l,\pm 1} \pm \frac{1}{k} \vec{\nabla} \times \left(j_l \vec{X}_{l,\pm 1} \right) \right]$$

$$c\vec{B}_0 = \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left(-\frac{i}{k} \vec{\nabla} \times \left(j_l \vec{X}_{l,\pm 1} \right) \pm i\, j_l \vec{X}_{l,\pm 1} \right)$$

(10.55)

Outgoing $A_l\, h_l^{(1)}(kr) \cdot \vec{X}_{l,\pm 1}$

spherically symmetric B.C. doesn't couple different m's.

$$\boxed{A_l = i^l \sqrt{4\pi(2l+1)} \cdot \tfrac{1}{2} \alpha_l}$$

5

$$\vec{E}_{sc} = \sum_{\ell=1}^{\infty} i^\ell \sqrt{4\pi(2\ell+1)} \Big[\frac{1}{2}\alpha_\ell^{\pm} h_\ell^{(1)}(kr) \vec{X}_{\ell,\pm 1} \pm \frac{1}{2}\beta_\ell \frac{1}{k} \vec{\nabla}\times\big(h_\ell \vec{X}_\ell\big)\Big]$$

$$c\vec{B}_{sc} = \sum_{\ell=1}^{\infty} i^\ell \sqrt{4\pi(2\ell+1)} \Big[-\frac{i}{k}\frac{1}{2}\alpha_\ell^{\pm} \vec{\nabla}\times\big(h_\ell \vec{X}_{\ell,\pm 1}\big) \mp i\beta_\ell^{\pm} h_\ell^{(1)}(kr) \vec{X}_{\ell,\pm 1}\Big]$$

(10.57)

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^*$$

$$P_{scatt} = \int_{r=a} d^2a \, \hat{r}\cdot \mathrm{Re}\Big[\frac{1}{2}\vec{E}\times\vec{H}^*\Big]$$

$$\longrightarrow \vec{E}\cdot(\vec{H}^*\times\hat{r}) = -\vec{E}_{sc}\cdot(\hat{r}\times\vec{H})_{sc}$$

$$P_{scatt} = -\frac{a^2}{2\mu_0}\int d\Omega \, \mathrm{Re}\big[\vec{E}_{sc}\cdot(\hat{r}\times\vec{B}_{sc})\big] \quad (10.58)$$

$$P_{abs} = \frac{a^2}{2\mu_0}\int d\Omega \, \mathrm{Re}\big[\vec{E}\cdot(\hat{r}\times\vec{B}^*)\big] \quad (10.59)$$

$$\vec{\nabla}\times\left(f\vec{X}\right) = \left(\frac{\partial f}{\partial r}+\frac{f}{r}\right)\hat{r}\times\vec{X} + i\,\cancel{\sqrt{l(l+1)}\,\frac{\hat{r}\,f\,Y_{lm}}{r}}$$

$$\vec{E}_{sc}\cdot\left(\hat{r}\times\vec{B}_{sc}\right) = \sum_{l,l'} (i)^{l}(-i)^{l'}\sqrt{4\pi(2l+1)}\sqrt{4\pi(2l'+1)}$$

$$\left(\frac{1}{2}\alpha_l h_l \vec{X}_l \pm \frac{1}{k}\frac{1}{2}\beta_l \vec{\nabla}\times\left(h_l\vec{X}_l\right)\right)$$

$$\cdot\left(\hat{r}\times\left[\frac{-i}{k}\frac{\alpha_{l'}}{2}\vec{\nabla}\times\left(h_{l'}\vec{X}_{l'}\right)\right.\right.$$

$$\left.\left.\mp\frac{1}{2}\beta_{l'}h_{l'}\vec{X}_{l'}\right)\right]$$

$\alpha_l \leftrightarrow \alpha_l'$

$\beta_l \leftrightarrow \beta_l'$

$$\sigma_{scatt} = \frac{\pi}{2k^2}\sum_{l=1}^{\infty}(2l+1)\left(|\alpha_l^{\pm}|^2+|\beta_l^{\pm}|^2\right)$$

$$\sigma_{abs} = \frac{\pi}{2k^2}\sum_{l=1}^{\infty}(2l+1)\left[2-|\alpha_l^{\pm}+1|^2-|\beta_l^{\pm}+1|^2\right]$$

$r\to\infty$

$$\vec{E}_{sc}\to\sum_{l} i^{l}\sqrt{4\pi(2l+1)}\,(-i)^{l+1}\frac{e^{ikr}}{r} \qquad (10.61)$$

$$\left(\frac{1}{2}\alpha_l \vec{X}_{l,\pm1} + i\frac{1}{2}\beta_l\hat{r}\times\vec{X}_{l,\pm1}\right)$$

$$\frac{d\sigma}{d\Omega} = \frac{r^2\frac{1}{2Z_0}|\vec{E}_{sc}|^2}{\frac{1}{2Z_0}|E_0|^2} = r^2|\vec{E}_{sc}|^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{scatt.} = \frac{\pi}{2k^2}\left|\sum_{l=1}^{\infty}\sqrt{2l+1}\left(\alpha_l^{\pm}\vec{X}_{l,\pm1}\pm i\beta_l\hat{r}\times\vec{X}_{l,\pm1}\right)\right|^2$$

(10.63)