3/28/2008

$S_1 = \{\text{circle of radius } \underline{a}\}$. $(ka \gg 1)$

$$\vec{k}_0 = k\left(\hat{x}\sin\theta_0\cos\phi_0 + \hat{y}\sin\theta_0\sin\phi_0 + \hat{z}\cos\theta_0\right)$$

$$\hat{r} = \left(\hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta\right)$$

$$\psi_{scatt} = \frac{-ik}{4\pi}\frac{e^{ikr}}{r}\int_0^a \underline{\rho' d\rho' d\phi'}\;\underline{(\cos\theta_0 + \cos\theta)}\, e^{i\vec{k}_0\cdot\vec{x}'}\, e^{-ik\hat{r}\cdot\vec{x}'}$$

$$\exp\Big[ik\rho'\sin\theta_0\left(\cos\phi_0\cos\phi' + \sin\phi_0\sin\phi'\right) - ik\rho'\sin\theta\left(\cos\phi\cos\phi' + \sin\phi\sin\phi'\right)\Big]$$

$$= \left[(\xi_x)\cos\phi' + (\xi_y)\sin\phi'\right](ik\rho')$$

$$= \vec{\xi}\cdot\hat{\rho}' = \xi\cdot\cos(\phi' - \delta).$$

$$\xi_x = \sin\theta_0\cos\phi_0 - \sin\theta\cos\phi$$

$$\xi_y = \sin\theta_0\sin\phi_0 - \sin\theta\sin\phi.$$

$$\xi^2 = \xi_x^2 + \xi_y^2 = \underline{\sin^2\theta_0 + \sin^2\theta - 2\sin\theta_0\sin\theta\cos(\phi - \phi_0)}.$$

$$\theta \ll 1 \rightarrow \theta_0^2 + \theta^2 - 2\theta\theta_0\cos(\phi - \phi_0)$$

$$= \underline{|\Delta\vec{\theta}|^2}.$$

$$\left.\begin{aligned}\xi_x &= \xi\cos\delta\\ \xi_y &= \xi\sin\delta\end{aligned}\right\}\; \tan\delta = \xi_y/\xi_x \quad \text{unnecessary.}$$

$$\psi(x) = \frac{-ik}{4\pi} \frac{e^{ikr}}{r} (\cos\theta + \cos\theta_0) \int_0^a \rho' d\rho' \int d\phi' \, e^{ik\rho'\xi \cos(\phi - \delta)}$$

$$= (\cdots) \int_0^a \rho' d\rho' \cdot 2\pi J_0(k\rho'\xi)$$

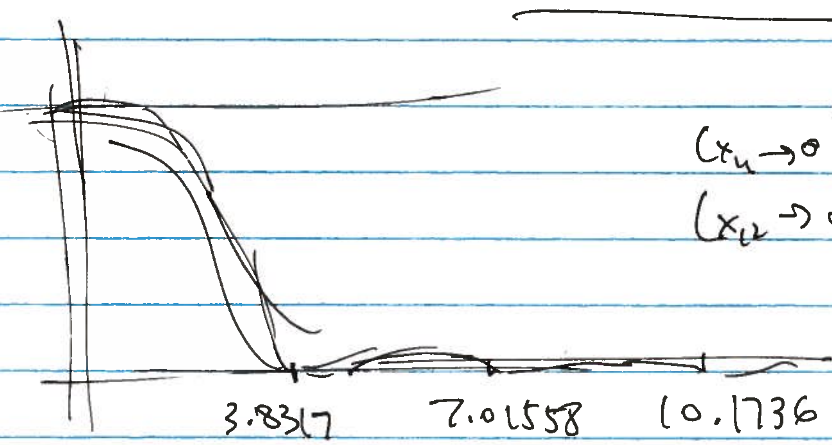
$$= (\cdots) \cdot \frac{2\pi a^2}{ka\xi} J_1(ka\xi).$$

$$\boxed{\psi = -i a^2 \left(\frac{\cos\theta + \cos\theta_0}{2} \right) \frac{J_1(ka\xi)}{ka\xi} \frac{e^{ikr}}{r}}$$

$$\boxed{|\psi|^2 = I = I_0 \left| \frac{J_1(ka\xi)}{ka\xi/2} \right|^2}$$

"Airy disk". $ka\xi = x_{11} = 3.8317 = \frac{2\pi}{\lambda} a \, \Delta\theta$

Diameter $= 2\Delta\theta = \frac{3.832}{\pi} \frac{\lambda}{a} = 1.21967 \frac{\lambda}{a}$.



$(x_{11} \to 0)$ 0.16225

$(x_{12} \to 0)$ 0.09007

Babinet's Principle.

Apertures (A) = Screen (B)

Screen (A) = Apertures (B).

$$\psi(x) = \frac{1}{4\pi} \int_{\text{Apertures}} da'^{2} \left(\psi_0 \frac{\partial G}{\partial n'} + G \frac{\partial \psi_0}{\partial n'} \right).$$

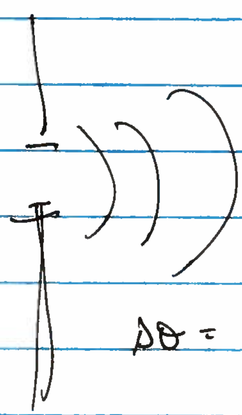
$$\psi_A = \frac{1}{4\pi} \int_{\text{Apertures}(A)} - - -$$

$$\psi_B = \frac{1}{4\pi} \int_{\text{Apertures}(B)} \cdots$$

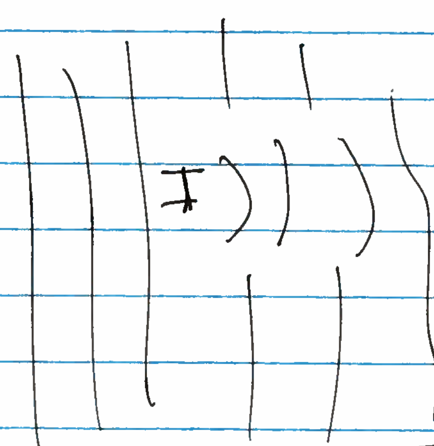
$$\psi_A + \psi_B = \int_{(\text{All})} \cdots = \psi_0 .$$

$$\psi_B = \psi_0 - \psi_A$$

§ 10.8



$\Delta\theta = \frac{\lambda}{a}$



Poisson's Bright Spot

vector Kirchhoff approximation. ⟶

vector Green's Theorem,

$$\int_V d^3x \left[\vec{F}\cdot(\nabla^2\vec{E}) - \vec{E}\cdot(\nabla^2\vec{F}) \right]$$

$$= \oint d^2a \Big[\hat{n}\cdot\left(\vec{F}(\nabla\cdot\vec{E}) - \vec{E}(\nabla\cdot\vec{F})\right)$$

$$- \left[\hat{n}\times(\nabla\times\vec{E}) \right]\cdot\vec{F} - (\hat{n}\times\vec{E})\cdot(\nabla\times\vec{F}) \Big]$$

$$\vec{E}(\vec{x}) = -\oint_S d^2a' \left[(\hat{n}'\cdot\vec{E})\nabla' G + (\hat{n}'\times(\nabla'\times\vec{E}))\, G \right.$$

$$\left. + (\hat{n}'\times\vec{E})\times\nabla' G \right].$$

$$G \to \frac{e^{ikR}}{4\pi R} \to \frac{e^{ikr}}{4\pi r}\, e^{-ik\hat{r}\cdot\vec{x}'}$$

$$\varepsilon^*\cdot\vec{F} = \frac{ik}{4\pi}\oint_{S_1} d^2a' \left[\underbrace{\vec{\varepsilon}^*\cdot(\hat{n}'\times\vec{B})}_{}\Big|_S + \vec{\varepsilon}^*\cdot\underline{(\hat{k}\times(\hat{n}'\times\vec{E}))}\Big|_S \right] e^{-ik\hat{r}\cdot\vec{x}'}$$

illuminated + shadow. $\vec{E}=0$, $\vec{B}=0$

$$\left. \underline{\vec{E}_{sc} = -\vec{E}_0}\,,\quad \underline{\vec{B}_{sc} = -\vec{B}_0} \right|_S.$$

$\Delta\vec{E}_{\parallel} = 0$. $\Delta\vec{B}_{\perp} = 0$

$\vec{E}_{sc} = -\vec{E}_0$, $\vec{B}_{sc} = +\vec{B}_0$

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conducting sphere. $\vec{E}_{\parallel}\big|_S = 0$ $\quad B_{\perp}\big|_S = 0$.

$\hookrightarrow$ shadow. $\vec{E}_{sc} = -\vec{E}_0$, $\vec{B}_{sc} = -\vec{B}_0$

$\vec{E} = \vec{B} = 0$

illuminated. $\vec{E}_{\parallel sc}\big|_S = -\vec{E}_{0\parallel}\big|_S$

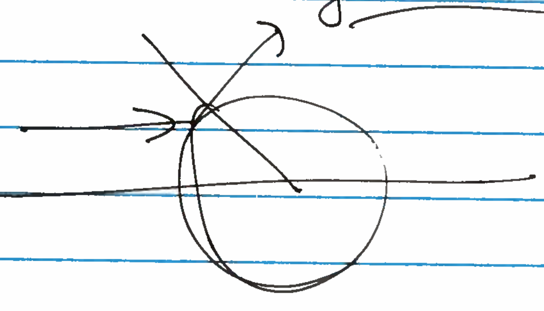
$\vec{B}_{\parallel, sc}\big|_S = +\vec{B}_{0,\parallel}\big|_S$

shadow Geometry $\rightarrow$ $\quad \vec{\varepsilon}^{\,*}\cdot\vec{f}_{sc} = \frac{ik}{2\pi}\left(\vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_0\right)\int_{\text{shadow}} d^2 a_\perp \cdot e^{-i\vec{k}_\perp\cdot\vec{x}_\perp}$

circular disk $\rightarrow$ $\boxed{2\pi a^2 \cdot \frac{J_1(ka\sin\theta)}{ka\sin\theta}}$ (just did)

$ka \to \infty$: $\delta(\theta)$

illuminated $\rightarrow$ geometric optics.



$\vec{\varepsilon}^{\,*}\cdot\vec{f}_{ill} \simeq \frac{a}{2}\,\vec{\varepsilon}^{\,*}\cdot\vec{\varepsilon}_r \; e^{-ika\cdot 2\sin\theta/2}$

reflected. $\vec{\varepsilon}_r = -\vec{\varepsilon}_0 + 2(\hat{n}\cdot\vec{\varepsilon}_0)\hat{n}$

$\vec{\varepsilon}_r\cdot\hat{n} = \vec{\varepsilon}_0\cdot\hat{n}$ $\qquad \vec{\varepsilon}_r\times\hat{n} = -\vec{\varepsilon}_0\times\hat{n}$