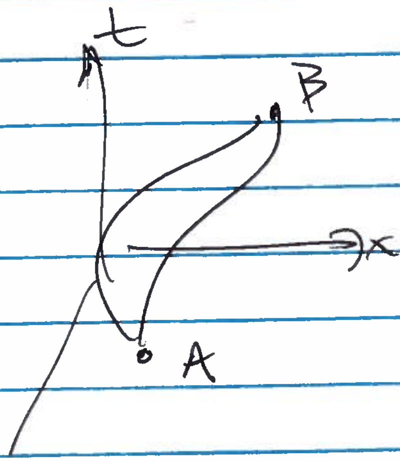
1/10/2018. **c same for all observers**

$\Longrightarrow$ time isn't same for all observers

$$d\tau^2 = \frac{dx\cdot dx}{c^2} = dt^2 - \frac{1}{c^2}(d\vec{x})^2 = dt^2\left(1-\frac{v^2}{c^2}\right)$$

scalar



$$\tau_{AB} = \int_A^B d\tau$$

$$= \int_A^B dt\left(1-\frac{v^2}{c^2}\right)^{1/2}$$

path dependent

Twin paradox

$$T_{(\alpha\beta\gamma)} \cdot \frac{(n+s-1)!}{(n-1)!\,s!} \qquad n=4,\ s=3 \qquad \frac{4\cdot 5\cdot 6}{3!} = 20$$

111 222 333 000 4

001 002 003
110 112 113 + 12
220 221 223
330 331 332

012 013 023 123 + 4

001

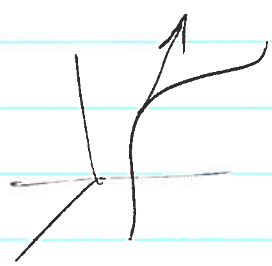
$$u^\mu = \frac{dx^\mu}{d\tau} = \begin{pmatrix} c\frac{dt}{d\tau} \\ \frac{d\vec{x}}{d\tau} \end{pmatrix} = \begin{pmatrix} c\cdot\gamma\frac{dt}{dt} \\ \gamma\frac{d\vec{x}}{dt} \end{pmatrix} = \begin{pmatrix} \gamma c \\ \gamma\vec{v} \end{pmatrix}$$

4-vector velocity $\boxed{\text{4-velocity}}$

$$u\cdot u = (\gamma c)^2 - (\gamma\vec{v})^2 = c^2\gamma^2\left(1-\frac{v^2}{c^2}\right)$$

$\boxed{u\cdot u = c^2}$ $c=1$. unit vector

u^μ = tangent to world-line



$a^\mu = \frac{du^\mu}{d\tau}$ 4-acceleration

$u\cdot u = c^2$

$$\frac{d}{d\tau}(u\cdot u) = 2u\cdot a = 0 \qquad \boxed{u\cdot a = 0}$$

at rest (proper frame, comoving frame)

$\vec{v} = 0$ $\quad u = \begin{pmatrix} c \\ \vec{0} \end{pmatrix}$

$a = \begin{pmatrix} 0 \\ \vec{a} \end{pmatrix}$ $\quad a\cdot a = -a^2_{\text{proper}}$

Constant proper acceleration $a_{\text{proper}} = g$.

$$u \cdot u = (u^t)^2 - (u^x)^2 = c^2$$

$$a \cdot a = (a^t)^2 - (a^x)^2 = -g^2$$

$$u \cdot a = (u^t)(a^t) - (u^x)(a^x) = 0.$$

$$(a^x)^2 = (a^t)^2 + g^2 = a^2 + g^2$$

$$(u^x)^2 = (u^t)^2 - c^2 = u^2 - c^2$$

$$(u^t)^2(a^t)^2 = u^2 a^2 = (u^x)^2(a^x)^2 = (u^2 - c^2)(a^2 + g^2)$$

$$\cancel{u^2 a^2} = \cancel{u^2 a^2} + u^2 g^2 - a^2 c^2 - c^2 g^2$$

$$\boxed{\frac{u^2}{c^2} - \frac{a^2}{g^2} = 1}$$

$$\left(\frac{du}{d\tau}\right)^2 = g^2\left(\frac{u^2}{c^2} - 1\right) \qquad \underline{u = c} \quad \text{at} \quad \underline{\tau = 0}$$

$$\int_c^u \frac{du'}{\sqrt{\frac{u'^2}{c^2} - 1}} = c \cdot \cosh^{-1}\left(\frac{u}{c}\right)\Big|_c^u = c \cdot \cosh^{-1} u = g\tau$$

$$\boxed{u^t = c \cdot \cosh\left(\frac{g\tau}{c}\right)} = c\frac{dt}{d\tau}.$$

$$\boxed{ct = \frac{c^2}{g} \cdot \sinh\left(\frac{g\tau}{c}\right)} \approx \frac{c^2}{g} \cdot \frac{g\tau}{c} = c\tau \qquad t \simeq \tau$$

$$(u^x)^2 = \left(\frac{dx}{d\tau}\right)^2 = (u^t)^2 - c^2 = c^2\left(1 - \cosh^2\frac{g\tau}{c}\right)$$

$$= c^2 \cdot \sinh^2\frac{g\tau}{c}$$

$$u^x = \frac{dx}{d\tau} = c \cdot \cancel{\sinh}\ \frac{g\tau}{c}$$

$$\boxed{x = \frac{c^2}{g} \cdot \left[\cosh\frac{g\tau}{c} - 1\right]} = \underline{\underline{\tfrac{1}{2} g t^2}}$$

$$\simeq \frac{c^2}{g}\left(\cancel{1} + \frac{1}{2}\left(\frac{g\tau}{c}\right)^2 - \cancel{1}\right) = \frac{1}{2} g \tau^2$$

$$\left(x + \frac{c^2}{g}\right)^2 - c^2 t^2 = \left(\cosh^2\frac{g\tau}{c} - \sinh^2\frac{g\tau}{c}\right) \cdot \left(\frac{c^2}{g}\right)^2 = \underline{\underline{\left(\frac{c^2}{g}\right)^2}}$$

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another way $\quad u = \Lambda u_0 \quad u^\mu = \exp(\zeta K)\begin{pmatrix} c \\ 0 \end{pmatrix}$

$$\left.\begin{aligned} (ct)' &= \gamma\left(ct + \frac{v}{c}x\right) \\ x' &= \gamma\left(x + \frac{v}{c}\cdot ct\right) \end{aligned}\right\} \begin{pmatrix} \cosh\zeta & \sinh\zeta \\ \sinh\zeta & \cosh\zeta \end{pmatrix}\begin{pmatrix} ct \\ x \end{pmatrix}$$

$\zeta = \zeta(\tau)$

$$\frac{du}{d\tau} = \dot{\zeta} K \cdot \exp(\zeta K)\cdot u^0$$

$$\text{———} = a = \exp(\zeta K)\, a^0 .$$

at $\zeta = 0$ $\quad \dot{\zeta}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\mathbb{1}\cdot\begin{pmatrix} c \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ g \end{pmatrix} = \dot{\zeta}\begin{pmatrix} 0 \\ c \end{pmatrix}$

$\dot{\zeta} = g/c \qquad \zeta = \frac{g\tau}{c}$

$$\boxed{\begin{aligned} u^t &= c\cdot\cosh\left(\frac{g\tau}{c}\right) \\ u^x &= c\cdot\sinh\left(\frac{g\tau}{c}\right) \end{aligned}}$$