4/9/2008

Constant acceleration.

$u_0 = \begin{pmatrix} c \\ \vec{0} \end{pmatrix}$ $\quad a_0 = \begin{pmatrix} 0 \\ \vec{g} \end{pmatrix}$

rocket frame is boosted. $\Lambda = \exp(\zeta K) = \exp\begin{pmatrix} 0 & \zeta \\ \zeta & 0 \end{pmatrix}$

$u = \Lambda u_0 = \exp(\zeta K)\, u_0$.

$a = \Lambda a_0 = \exp(\zeta K)\, a_0$

$u = \frac{dq}{d\tau}$ $\quad \Lambda a_0 = \dot{\zeta} K \cdot \exp(\zeta K)\, u_0$

$\Lambda \begin{pmatrix} 0 \\ g \end{pmatrix} = \dot{\zeta} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Lambda \begin{pmatrix} c \\ 0 \end{pmatrix}$

Λ^{-1} $\zeta = 0$ $\rightarrow \begin{pmatrix} 0 \\ g \end{pmatrix} = \dot{\zeta} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c \\ 0 \end{pmatrix} = \dot{\zeta} \begin{pmatrix} 0 \\ c \end{pmatrix}$

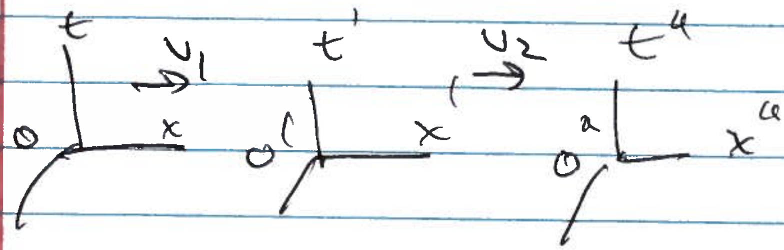
$\dot{\zeta} = g/c$. $\zeta = \frac{g\tau}{c}$ $\quad u = \exp\left(\frac{g\tau}{c} K\right)$

$u = \begin{pmatrix} \cosh\zeta & \sinh\zeta \\ \sinh\zeta & \cosh\zeta \end{pmatrix} \begin{pmatrix} c \\ 0 \end{pmatrix}$

$u^t = c \cdot \cosh \frac{g\tau}{c}$ $\quad ct = \frac{c^2}{g} \sinh\left(\frac{g\tau}{c}\right)$

$u^x = c \cdot \sinh \frac{g\tau}{c}$ $\quad x = \frac{c^2}{g} \left(\cosh \frac{g\tau}{c} - 1\right)$

$d\tau^2 = \left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dx}{d\tau}\right)^2 = \left(\cosh^2 \frac{g\tau}{c}\right) d\tau^2 - \sinh^2 \frac{g\tau}{c}\right) d\tau^2 = d\tau^2$



$x' = \Lambda_1 x$ $\qquad$ $x'' = \Lambda_2 x'$

$\hookrightarrow \; x'' = \Lambda_2 \Lambda_1 x \; = \Lambda x.$

$$\Lambda = \Lambda_2 \Lambda_1 = \begin{pmatrix} \gamma_2 & \gamma_2 \frac{v_2}{c} \\ \gamma_2 \frac{v_2}{c} & \gamma_2 \end{pmatrix} \begin{pmatrix} \gamma_1 & \gamma_1 \frac{v_1}{c} \\ \gamma_1 \frac{v_1}{c} & \gamma_1 \end{pmatrix} = \begin{pmatrix} \gamma & \gamma \frac{v}{c} \\ \gamma \frac{v}{c} & \gamma \end{pmatrix}$$

$$= \begin{pmatrix} \gamma_1 \gamma_2 \left(1 + \frac{v_2 v_2}{c^2}\right) & \gamma_1 \gamma_2 \left(\frac{v_1}{c} + \frac{v_2}{c}\right) \\ \gamma_1 \gamma_2 \left(\frac{v_1}{c} + \frac{v_2}{c}\right) & \gamma_1 \gamma_2 \left(1 + \frac{v_1 v_2}{c^2}\right) \end{pmatrix}$$

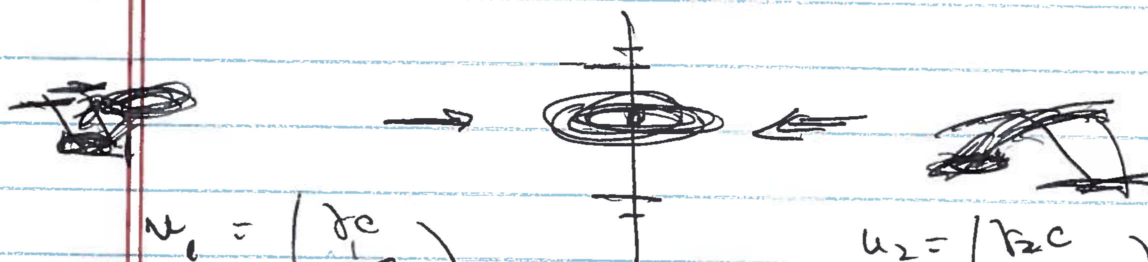
$$\frac{\gamma \frac{v}{c}}{\gamma} = \frac{\cancel{\gamma_1 \gamma_2} \left(\frac{v_1}{c} + \frac{v_2}{c}\right)}{\cancel{\gamma_1 \gamma_2} \left(1 + \frac{v_1 v_2}{c^2}\right)} = \frac{v}{c} = \frac{\frac{v_1}{c} + \frac{v_2}{c}}{1 + \frac{v_1 v_2}{c^2}}$$

$$\boxed{v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}}$$

independent of order.

$$\exp(\varsigma_1 K) \exp(\varsigma_2 K) = \exp\left((\varsigma_1 + \varsigma_2) K\right)$$

$$\frac{v}{c} = \tanh(\varsigma_1 + \varsigma_2) = \frac{\tanh \varsigma_1 + \tanh \varsigma_2}{1 + \tanh \varsigma_1 \tanh \varsigma_2}$$



$u_1 = \begin{pmatrix} \gamma_1 c \\ \gamma_1 \vec{v}_1 \end{pmatrix}$ $\qquad$ $u_2 = \begin{pmatrix} \gamma_2 c \\ \gamma_2 \vec{v}_2 \end{pmatrix}$

as seen from (DS9)

As seen from (NCC1701). $\quad u_1 = \begin{pmatrix} c \\ \vec{0} \end{pmatrix}$

$u_2 = \begin{pmatrix} \gamma c \\ \gamma \vec{v} \end{pmatrix}$ relative

$$u_1 \cdot u_2 = \gamma c \cdot c - \cancel{(\vec{0} \cdot \gamma \vec{v})} = \underline{\gamma c^2}$$

$$u_1 \cdot u_2 = (\gamma_1 c)(\gamma_2 c) - (\gamma_1 \vec{v}_1) \cdot (\gamma_2 \vec{v}_2)$$

$$= \gamma_1 \gamma_2 c^2 \left(1 - \frac{\vec{v}_1 \cdot \vec{v}_2}{c^2}\right)$$

$$\boxed{\gamma_{rel} = \gamma_1 \gamma_2 \left(1 - \frac{\vec{v}_1 \cdot \vec{v}_2}{c^2}\right)}$$

colinear, headon $\rightarrow$ $\gamma = \gamma_1 \gamma_2 \left(1 + \frac{v_1 v_2}{c^2}\right)$

(4)

More 4-vectors

$\vec{p} = m\vec{v}$. $p^\mu = m u^\mu$??

$$p^\mu = \begin{pmatrix} \gamma m c \\ \gamma m \vec{v} \end{pmatrix} \leftarrow \text{relativistic momentum}$$

$v \ll c$ $\quad \gamma m c \simeq mc\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) = \frac{1}{c}\left(mc^2 + \frac{1}{2}mv^2 + \cdots\right)$

$\uparrow$ rest energy $\quad \uparrow$ K.E.

Identify $\quad p^i = \vec{p} = \gamma m \vec{v}$

$p^0 = \frac{E}{c} = \frac{\gamma m c^2}{c}$ $\qquad \boxed{E = \gamma m c^2}$

$p \cdot p = m^2\, u \cdot u = m^2 (c^2)$

$\left(\frac{E}{c}\right)^2 - |\vec{p}|^2 = m^2c^2 \qquad \boxed{E^2 = (c\vec{p})^2 + (mc^2)^2}$

~~"$m_{rel} = \gamma m = \frac{m}{\sqrt{1-v^2/c^2}}$"~~ No $\qquad$ ~~ict~~ No

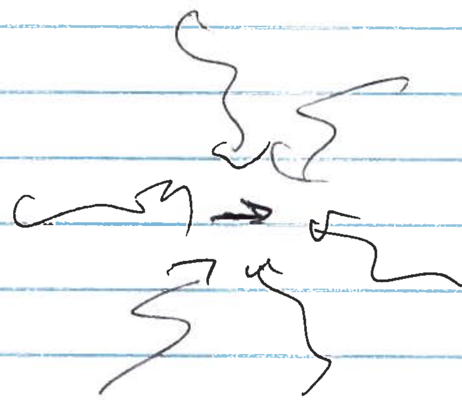
$v \to c$, $\gamma \to \infty$ $\quad m \to 0$: $(\gamma m c^2) = E$ fixed.

$\boxed{E^2 = |c\vec{p}|^2}$ massless. photon. $\quad E = |\vec{p}c|$

$E = \hbar\omega \qquad \vec{p} = \hbar\vec{k} \qquad p^\mu = \hbar k^\mu \qquad k^\mu = \begin{pmatrix} \omega/c \\ \vec{k} \end{pmatrix}$

$$k\cdot x = (k^0)(x^0) - \vec{k}\cdot\vec{x} = \left(\frac{\omega}{c}\right)(ct) - \vec{k}\cdot\vec{x}$$

$$e^{-ik\cdot x} = e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$ phase is Lorentz invariant.



Isotropic Thermal bath

moving observer $u^\mu = \begin{pmatrix}\gamma c \\ \gamma\vec{v}\end{pmatrix}$

$p^\mu = \hbar k^\mu = \begin{pmatrix}\hbar\omega/c \\ -\hbar k\hat{n}\end{pmatrix}$

$$u\cdot p = (c)\left(\frac{\hbar\omega_{obs}}{c}\right) = (\gamma c)\left(\frac{\hbar\omega}{c}\right) - (\gamma\vec{v})\cdot(\hbar\vec{k})$$

$$= \hbar\omega\,\gamma\,\left(1+\frac{\hat{n}\cdot\vec{v}}{c}\right)$$

$$\omega_{obs} = \gamma\omega\left(1+\frac{\hat{n}\cdot\vec{v}}{c}\right)$$

$$\omega_{obs} = \gamma\omega\left(1+\frac{v}{c}\cos\theta\right)$$

not θ_{obs}

$$\omega_{obs} = \frac{\omega}{\gamma_{obs}\left(1-\frac{v}{c}\cos\theta_{obs}\right)}$$

$$T_{obs} \simeq \frac{T_0}{\gamma\left(1-\frac{v}{c}\cos\theta_{obs}\right)} \simeq T_0\left(1+\frac{v}{c}\cos\theta_{obs}\right).$$

Electromagnetism.

Q. (electric charge) : tentatively scalar.

$\frac{v}{c} = \frac{1}{137}$, $(6\times10^{23}\,\text{cm}^{-3})(1.6\times10^{-19}\,\text{C})\left(\frac{1}{2}\frac{1}{137^2}\right)$

$$= \frac{(96{,}500\,\text{C})}{2\cdot 137^2}\,\text{cm}^{-3} = \frac{96{,}500}{37538} = \underline{\underline{2.56\ \text{C}/\text{cm}^3}}$$

$\rho = \frac{dQ}{dV}$ $\qquad d^4x' = dt'dx'dy'dz' = \left|\det \frac{\partial x'}{\partial x}\right| d^4x$

$$= \underbrace{|\det \Lambda|}_{1}\, dt\, dx\, dy\, dz\ .$$

$\underline{\rho = \frac{dQ}{dx\,dy\,dz}}$ is $\underline{\text{time}}$-component of a $\underline{\text{vector}}$

$$\underline{d^3\Sigma_\mu = \varepsilon_{\mu\alpha\beta\gamma}\, dx^\alpha dx^\beta dx^\gamma}$$

$$\underline{\text{vol} = \vec{a}\cdot(\vec{b}\times\vec{c})}$$

$\underline{J_z = \frac{dQ}{dt\,dx\,dy}}$: z-component of a vector.

$\boxed{J^\mu = \begin{pmatrix} c\rho \\ \vec{J} \end{pmatrix}}$ 4-vector current density

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = \cancel{\tfrac{\partial}{\partial t}}\ \frac{1}{c}\frac{\partial}{\partial t}(c\rho) + \vec{\nabla}\cdot\vec{J} = \frac{\partial}{\partial x^0}J^0 + \frac{\partial}{\partial x^i}J^i$$

$\boxed{\nabla_\mu J^\mu = 0}$ $\underline{\text{equation of continuity}}$ is relativistic $\underline{\text{invariant}}$!!

$$\Box^2 = \vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = -\eta^{\alpha\beta}\,\partial_\alpha \partial_\beta$$

$$\boxed{\Box^2 \Phi = (-\rho/\varepsilon_0)} \qquad \boxed{\Box^2 \vec{A} = (-\mu_0 \vec{J})}$$

$$\Rightarrow \boxed{A^\mu = \left(\begin{matrix}\Phi/c \\ \vec{A}\end{matrix}\right)} \qquad \text{But First}$$

Gaussian Units

2 changes plus one rule.

rule: remove μ_0 in favor of $\varepsilon_0 c^2$.

$$\mu_0 \varepsilon_0 = \frac{1}{c^2} \qquad \boxed{\mu_0 = \frac{1}{\varepsilon_0 c^2}}$$

change: $(c\vec{B}_{SI}) \to \vec{B}_{Gaussian}$

$$\underbrace{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}_{\text{was}} = -\frac{1}{c}\frac{\partial}{\partial t}(c\vec{B}) \longrightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}}\ \text{Gaussian}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\cdot(c\vec{B}) = 0 \qquad \boxed{\vec{\nabla}\cdot\vec{B} = 0}\ \text{Gaussian}$$

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) = q\left(\vec{E} + \frac{\vec{v}}{c}\times c\vec{B}\right)$$

$$\boxed{\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}_{Gauss}\right)}\ \text{Gaussian}$$

change units
of charge so $\boxed{4\pi\varepsilon_0 = 1}$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2} = \left(\frac{Q_1}{\sqrt{4\pi\varepsilon_0}}\right)\left(\frac{Q_2}{\sqrt{4\pi\varepsilon_0}}\right)\frac{1}{r^2}$$

gaussian $\boxed{F = \frac{Q_1' Q_2'}{r^2}}$

[coulomb] $\rightarrow$ [e.s.u.] $\boxed{e = 4.803 \times 10^{-10} \text{ esu}}$

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$. $\boxed{\vec{\nabla}\cdot\vec{E} = 4\pi\rho}$
Gaussian

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$= \frac{1}{c^2\varepsilon_0}\vec{J} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}.$$

$$\vec{\nabla}\times(c\vec{B}) = \frac{1}{c\varepsilon_0}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}.$$

$$\boxed{\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}}$$ Gaussian.

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{\mu_0\varepsilon_0}{\varepsilon_0^2}} = \sqrt{\frac{1}{c^2}(4\pi)^2} = \frac{4\pi}{c}.$$