4/11/2018

Gaussian Units

Jackson. Appendix 3 / Appendix 4, Table 2 / Table 3

Verso. Chapter 11 - 9.

First: remove μ_0 in favor of ε_0: $\mu_0 = \frac{1}{\varepsilon_0 c^2}$

$\vec{B}'_{gaussian} = (c\vec{B})_{SI}$

$[B']_G = [E']_G$.

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{1}{c} \cdot \frac{\partial}{\partial t}(c\vec{B})$

$\vec{\nabla} \times \vec{E}' = -\frac{1}{c}\frac{\partial \vec{B}'}{\partial t}$

$\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \cdot (c\vec{B}) = 0$

$\vec{\nabla} \cdot \vec{B}' = 0$

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = q(\vec{E} + \frac{\vec{v}}{c} \times (c\vec{B}))$

$\vec{F}' = q'(\vec{E}' + \frac{\vec{v}}{c} \times \vec{B}')$

2

change units of charge so $4\pi\varepsilon_0 = 1$

$\varepsilon_0 = \frac{1}{4\pi}$

~~Table~~ 2 p. 781 Line 3 wrong

$\varepsilon_0 = 1$ "Lorentz = Heaviside"

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2} = \left(\frac{Q_1}{\sqrt{4\pi\varepsilon_0}}\right)\left(\frac{Q_2}{\sqrt{4\pi\varepsilon_0}}\right)\frac{1}{r^2}$$

gaussian $F = \frac{Q_1' Q_2'}{r^2}$

[coulombs] $\rightarrow$ [e.s.u.] $e = 4.803 \times 10^{-10}$ esu

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 . \qquad \vec{\nabla}\cdot\vec{E}' = 4\pi\rho$$

Gaussian

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$= \frac{1}{c^2\varepsilon_0}\vec{J} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}.$$

$$\vec{\nabla}\times(c\vec{B}) = \frac{1}{c\varepsilon_0}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}.$$

$$\vec{\nabla}\times\vec{B}' = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}'}{\partial t}$$

Gaussian.

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{\mu_0\varepsilon_0}{\varepsilon_0^2}} = \sqrt{\frac{1}{c^2}(4\pi)^2} = \frac{4\pi}{c}.$$

In sum:

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \longrightarrow \quad \vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \quad \longrightarrow \quad \boxed{\vec{E} + \frac{1}{c}\frac{\partial\vec{A}}{\partial t} = -\vec{\nabla}\Phi}$$

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}. \qquad \boxed{\vec{E} = -\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}}$$

Above, below (11.141)

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\cdot\vec{A}$$

$$= \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial}{\partial t}\left(-\frac{\partial}{\partial t}\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial^2\vec{A}}{\partial t^2}\right)$$

$$\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \vec{\nabla}^2\vec{A} = \frac{4\pi}{c}\vec{J} - \vec{\nabla}\left(\frac{1}{c}\frac{\partial\Phi}{\partial t} + \vec{\nabla}\cdot\vec{A}\right)$$

Lorentz gauge: $\boxed{\frac{1}{c}\frac{\partial\Phi}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0}$ (11.133)

$$\partial_\mu A^\mu = 0$$

$$\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\cdot\left(-\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right) = -\vec{\nabla}^2\Phi - \frac{1}{c}\vec{\nabla}\cdot\left(\frac{\partial \vec{A}}{\partial t}\right) = 4\pi\rho$$

$$-\frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A})$$

$$= +\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}$$

$$\boxed{\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} - \nabla^2\Phi = 4\pi\rho = \frac{4\pi}{c}(c\rho)}$$

$$A^\mu = \begin{pmatrix}\Phi \\ \vec{A}\end{pmatrix} \qquad \boxed{\Box^2 A^\mu = \frac{4\pi}{c}J^\mu}$$

4-vector potential = vector potential.

$\vec{E}, \vec{B}$?? . fourth, scalar object (twice?) related to each other.

$\vec{B} = \vec{\nabla}\times\vec{A}$. $\quad$ ~~$B^x = \partial_y A^z - \partial_z A^y$~~

$$B^x = \frac{\partial A^y}{\partial z} - \frac{\partial A^z}{\partial y} \qquad (y, z)$$

$$E^x = -\frac{\partial\Phi}{\partial x} - \frac{1}{c}\frac{\partial A^x}{\partial t} \qquad (x, t).$$

signs differ, η contains signs —

Look at $\boxed{F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu}$

F is antisymmetric. $F^{\mu\nu} = -F^{\nu\mu}$

$\frac{4\times 3}{2} = 6$ degrees of freedom.

$$F^{01} = \partial^0 A^1 - \partial^1 A^0 = (\eta^{00})\left(\frac{1}{c}\frac{\partial}{\partial t}\right)(A^x) - (\eta^{11})\left(\frac{\partial}{\partial x}\right)(\Phi)$$

$$= \frac{1}{c}\frac{\partial A^x}{\partial t} + \frac{\partial \Phi}{\partial x} = -(E^x) = -F^{10} \qquad \text{etc.}$$

$$F^{23} = \partial^2 A^3 - \partial^3 A^2 = (\eta^{22})\left(\frac{\partial}{\partial y}\right)(A^z) - (\eta^{33})\left(\frac{\partial}{\partial z}\right)(A^y)$$

$$= -\frac{\partial A^z}{\partial y} + \frac{\partial A^y}{\partial z} = -(B^x) = -F^{32}.$$

$$\boxed{F^{\alpha\beta} = \begin{pmatrix} 0 & -E^x & -E^y & -E^z \\ E^x & 0 & -B^z & B^y \\ E^y & B^z & 0 & -B^x \\ E^z & -B^y & B^x & 0 \end{pmatrix}}$$

(11.137)

(I always have to look up signs or work out again every time)

$$F_{\alpha\beta} = \eta_{\alpha\mu}\eta_{\beta\nu}F^{\mu\nu} = (\eta_{\alpha\mu})(F^{\mu\nu})(\eta^T_{\nu\beta})$$

$$= (\eta F \eta^T). = \left(\begin{array}{c|ccc} 1 & & & \\ \hline & -1 & & \\ & & -1 & \\ & & & -1 \end{array}\right)\left(\;F\;\right)\left(\begin{array}{c|ccc} 1 & & & \\ \hline & -1 & & \\ & & -1 & \\ & & & -1 \end{array}\right)$$

change sign of each i-row, each j-column.

each $\underline{B}$ reverses sign twice. $\left.\right\}$ $\underline{B} \to \underline{B}$

each $\underline{E}$ reverses sign once $\quad \underline{E} \to -\underline{E}$

(11.138)

<u>Dual</u> (again). $\quad \varepsilon^{0123} = +1$

$$*F^{\alpha\beta} = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu}F_{\mu\nu} = \mathcal{F}^{\alpha\beta}.$$

$$*F^{01} = \frac{1}{2}\varepsilon^{01\mu\nu}F_{\mu\nu} = \frac{1}{2}\varepsilon^{0123}F_{23} + \frac{1}{2}\varepsilon^{0132}F_{32}$$

$$= \frac{1}{2}(+1)(-B^x) + \frac{1}{2}(-1)(+B^x) = \underline{-B^x}$$

$$*F^{23} = \frac{1}{2}\varepsilon^{23\mu\nu}F_{\mu\nu} = \frac{1}{2}\varepsilon^{2301}F_{01} + \frac{1}{2}\varepsilon^{2310}F_{10}$$

$$= \frac{1}{2}(+1)(E^x) + \frac{1}{2}(-1)(-E^x) = E^x.$$

$$
{}^{*}F^{\alpha\beta} = \begin{pmatrix} 0 & -B^x & -B^y & -B^z \\ B^x & 0 & E^z & -E^y \\ B^y & -E^z & 0 & E^x \\ B^z & E^y & -E^x & 0 \end{pmatrix}
$$

(A.140)

$\vec{E} \rightarrow \vec{B}$

$\vec{B} \rightarrow -\vec{E}$

$\underline{\text{Dual}}_1 = \underline{\text{Dual}}_2$