4/13/2017

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E^x & -E^y & -E^z \\ E^x & 0 & -B^z & B^y \\ E^y & B^z & 0 & -B^x \\ E^z & -B^y & B^x & 0 \end{pmatrix}$$

$$F_{\alpha\beta} = \begin{pmatrix} 0 & E^x & E^y & E^z \\ -E^x & 0 & -B^z & B^y \\ -E^y & B^z & 0 & -B^x \\ -E^z & -B^y & B^x & 0 \end{pmatrix}$$

$\vec{E} \to -\vec{E}$

$\vec{B} \to \vec{B}$

$$^*F^{\alpha\beta} = \begin{pmatrix} 0 & -B^x & -B^y & -B^z \\ B^x & 0 & E^z & -E^y \\ B^y & -E^z & 0 & E^x \\ B^z & E^y & -E^x & 0 \end{pmatrix}$$

$= \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} F_{\mu\nu}$

$\vec{E} \to \vec{B}$

$\vec{B} \to -\vec{E}$

$$F_{\mu\nu} F^{\mu\nu} = -2\vec{E}\cdot\vec{E} + 2\vec{B}\cdot\vec{B} = 2(B^2 - E^2)$$

$$^*F^{\mu\nu} F_{\mu\nu} = -4\vec{B}\cdot\vec{E}$$

Invariants!

D

Lorentz transformation

$$F'^{\alpha\beta} = \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu F^{\mu\nu} = (\Lambda)^\alpha{}_\mu (F)^{\mu\nu} (\Lambda^T)_\nu{}^\beta$$

$$F' = \Lambda F \Lambda^T$$

x-boost.

$$F' = \left(\begin{array}{cc|cc} \gamma & \frac{\gamma v}{c} & & \\ \frac{\gamma v}{c} & \gamma & & \\ \hline & & 1 & \\ & & & 1 \end{array}\right) \left(\begin{array}{cc|cc} 0 & -E^x & -E^y & -E^z \\ E^x & 0 & -B^z & B^y \\ \hline E^y & B^z & 0 & -B^x \\ E^z & -B^y & B^x & 0 \end{array}\right) \left(\cdots \right)$$

$$= \left(\begin{array}{cc|cc} 0 & -\gamma^2(1-v^2/c^2)E^x & -\gamma E^y - \frac{\gamma v}{c}B^z & -\gamma E^z + \frac{\gamma v}{c}B^y \\ \gamma^2(1-\frac{v^2}{c^2})E^x & 0 & -\gamma B^z - \frac{\gamma v}{c}E^y & +\gamma B^y - \frac{\gamma v}{c}E^z \\ \hline \gamma E^y + \frac{\gamma v}{c}B^z & \gamma B^z + \gamma\frac{v}{c}E^y & 0 & -B^x \\ \gamma E^z - \frac{\gamma v}{c}B^y & -\gamma B^y + \gamma\frac{v}{c}E^z & B^x & 0 \end{array}\right)$$

$B'^x = B^x$ $\quad E'^x = E^x$ $\quad E'_\parallel = E_\parallel$ $\quad B'_\parallel = B_\parallel$

$$\vec{E}'_\perp = \gamma\left(\vec{E}_\perp + \frac{\vec{v}}{c}\times\vec{B}_\perp\right)$$ ← (newtonian $F'=F$)

$$\vec{B}'_\perp = \gamma\left(\vec{B}_\perp - \frac{\vec{v}}{c}\times\vec{E}_\perp\right)$$

(11.148) components

(11.149) vectors

Inhomogeneous Maxwell Equations.

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho = \frac{4\pi}{c}\cdot c\rho$$

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

sum of three derivatives of E

E^x, E^y, E^z all in same column (row)

B^y, B^z, E^x all in same column (row)

$\partial_\alpha F^{\alpha\beta}$

$$\partial_\alpha F^{\alpha 0} = \cancel{\partial_0 F^{00}} + \partial_1 F^{10} + \partial_2 F^{20} + \partial^3 F^{30}$$

$$= \left(\frac{\partial}{\partial x}\right)(E^x) + \left(\frac{\partial}{\partial y}\right)(E^y) + \left(\frac{\partial}{\partial z}\right)(E^z)$$

$$= \vec{\nabla}\cdot\vec{E} = \frac{4\pi}{c}\cdot c\rho = \frac{4\pi}{c}\cdot J^0$$

$$\partial_\alpha F^{\alpha 1} = \partial_0 F^{01} + \cancel{\partial_1 F^{11}} + \partial_2 F^{21} + \partial_3 F^{31}$$

$$= \left(\frac{1}{c}\frac{\partial}{\partial t}\right)(-E^x) + \left(\frac{\partial}{\partial y}\right)(B^z) + \left(\frac{\partial}{\partial z}\right)(-B^y)$$

$$= \left(\frac{\partial B^z}{\partial y} - \frac{\partial B^y}{\partial z} - \frac{1}{c}\frac{\partial E^x}{\partial t}\right) = \left(\vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t}\right)^x = \frac{4\pi}{c}J^x$$

$$\boxed{\partial_\alpha F^{\alpha\beta} = \frac{4\pi}{c} J^\alpha} \quad (11.14).$$

④

Homogeneous equations: $\vec{E} \to \vec{B}$, $\vec{B} \to -\vec{E}$

$$\boxed{\partial_\alpha \left({}^*F^{\alpha\beta}\right) = 0} \quad (11.145).$$

$$\partial_\alpha \left(\frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} F_{\mu\nu}\right) = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu}\left(\partial_\alpha F_{\mu\nu}\right) = 0$$

$$\varepsilon^{\mu\alpha\beta\gamma}\left(\partial_\alpha F_{\beta\gamma}\right) = 0 \qquad \boxed{\partial_{[\alpha} F_{\beta\gamma]} = 0}$$

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta}$$

$$-\partial_\alpha F_{\gamma\beta} - \partial_\beta F_{\alpha\gamma} - \partial_\gamma F_{\beta\alpha} = 0$$

$$\boxed{\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0} \quad (11.143.$$

Lorentz Force : $\vec{F} = \frac{d\vec{p}}{dt} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$

$$\gamma \frac{d\vec{p}}{dt} = \frac{d\vec{p}}{d\tau} = q\left(\gamma \vec{E} + \gamma \frac{\vec{v}}{c} \times \vec{B}\right) \leftarrow u^\beta \text{ contracted with } F^{\alpha\beta}$$

$$F^{\alpha\beta} u_\beta$$

$\alpha = 1$ $$F^{1\beta} u_\beta = F^{10} u_0 + \cancel{F^{11}} u_1 + F^{12} u_2 + F^{13} u_3$$

$$= (E^x)(\gamma c) + (-B^z)(-\gamma v^y) + (+B^y)(-\gamma v^z)$$

$$= \gamma c \left(E^x + \frac{v^y}{c} B^z - \frac{v^z}{c} B^y\right) = \gamma c\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

$$\boxed{\frac{dp^i}{d\tau} = \frac{q}{c} F^{i\beta} u_\beta} \quad (11.125)$$

$\alpha = 0$ $$F^{0\beta} u_\beta = \cancel{F^{00}} u_0 + F^{01} u_1 + F^{02} u_2 + F^{03} u_3$$

$$= (-\vec{E}) \cdot \left(-\frac{\vec{v}}{c}\right) = \frac{\vec{v}}{c} \cdot \vec{E}$$

$$\gamma \frac{dE}{dt} = \gamma \vec{F} \cdot \vec{v} = \gamma q \vec{E} \cdot \vec{v}$$

$$\frac{d}{d\tau}\left(\frac{E}{c}\right) = \frac{q}{c} \gamma \vec{v} \cdot \vec{E} \qquad \boxed{\frac{dp^0}{d\tau} = \frac{q}{c} F^{0\beta} u_\beta}$$

6

$$\boxed{\frac{dp^\alpha}{d\tau} = \frac{q}{c} F^{\alpha\beta} u_\beta} \quad (11.126).$$

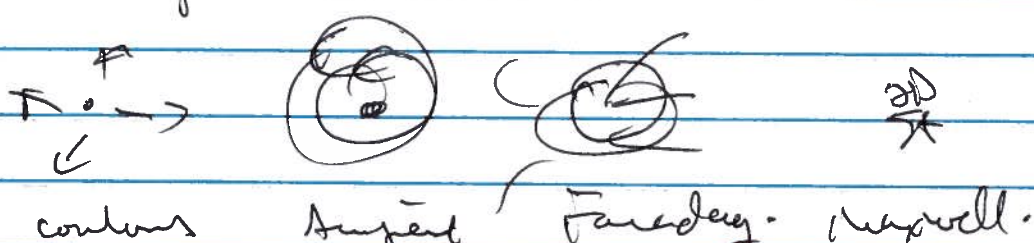
$$\frac{dp^\alpha}{d\tau} = m\frac{du^\alpha}{d\tau} = m a^\alpha.$$

antisymmetric

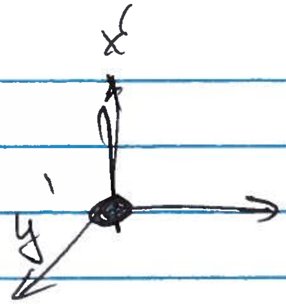
$$\cancel{u^\alpha a_\alpha}\; u^\alpha a_\alpha = u\cdot a = \frac{q}{mc} F^{\alpha\beta} u_\alpha u_\beta = 0$$

symmetric

Nothing extra,
nothing missing.



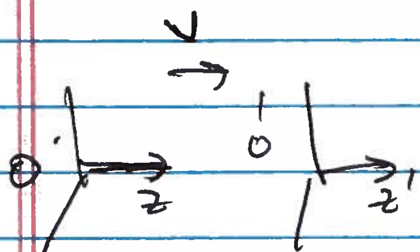
Point q



@ rest.

$$\vec{E}' = \frac{q\hat{r}'}{r'^2} = q\frac{\vec{x}'}{(r'^2)^{3/2}}$$

$$= q\left(\frac{x'\hat{x}' + y'\hat{y}' + z'\hat{z}'}{(x'^2+y'^2+z'^2)^{3/2}}\right)$$



$$x = x'$$
$$y = y'$$
$$z = \gamma(z' + vt')$$
$$t = \gamma\left(t' + \frac{z'v}{c^2}\right)$$

inverse.

$$z' = \gamma(z - vt)$$
$$t' = \gamma\left(t - \frac{vz}{c^2}\right)$$

origins coincide @ $t = t' = 0$

$$E^z = E_\parallel = E'_\parallel = E'^{z'} = \frac{q\,z'}{(x'^2+y'^2+z'^2)^{3/2}} = \frac{q\cdot\gamma(z-vt)}{[x^2+y^2+\gamma^2(z-vt)^2]^{3/2}}$$

$$\vec{E}_\perp = \gamma\left(\vec{E}'_\perp + \frac{\vec{v}}{c}\times\cancel{\vec{B}'_\perp}\right) = \gamma\vec{E}'_\perp = \gamma\cdot\frac{q\,\vec{x}'_\perp}{(r')^3}.$$

$$E^x = \frac{\gamma q x}{(x^2+y^2+\gamma^2(z-vt)^2)^{3/2}}$$

E^y ______ ..

@ $t = 0$

$$\boxed{\vec{E} = \frac{\gamma q(x\hat{x} + y\hat{y} + z\hat{z})}{(x^2+y^2+\gamma^2 z^2)^{3/2}}}$$

$z = r\cos\theta$

$$\vec{E} = \frac{\gamma q\,\vec{x}}{r^3(\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \gamma^2\cos^2\theta)^{3/2}}.$$

$$\vec{E} = \frac{\gamma q \hat{r}}{r^2} \frac{1}{(\sin^2\theta + \gamma^2\cos^2\theta)^{3/2}}$$

$$\vec{E} = \frac{q\hat{r}}{r^2}\frac{1}{\gamma^2}\frac{1}{\left((1-v^2/c^2)\sin^2\theta + \cos^2\theta\right)^{3/2}}$$

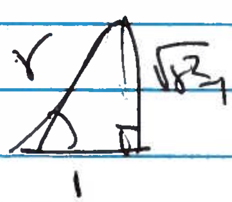
$$\boxed{\vec{E} = \frac{q\hat{r}}{r^2}\cdot\frac{1}{\gamma^2}\frac{1}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}}$$

radial — inverse square — squeezed in direction of motion

$$\oint_{\text{sphere}} d^2a \cdot \vec{E}\cdot\hat{r} = \int r^2 d\Omega \cdot \frac{q}{r^2}\cdot\frac{1}{\gamma^2}\frac{1}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$

$$= q\int d\Omega \frac{\gamma}{\left(1+(\gamma^2-1)\cos^2\theta\right)^{3/2}} = 2\pi\gamma\cdot\int_{-1}^{1} d\mu \frac{1}{\left(1+(\gamma^2-1)\mu^2\right)^{3/2}}$$

$$(\gamma^2-1)\mu^2 = \tan^2\phi \quad\to\quad 2\pi\gamma\int_{-\phi_0}^{\phi_0}\frac{\sec^{\cancel{2}}\phi\, d\phi}{\sqrt{\gamma^2-1}}\cdot\frac{1}{\cancel{(1+\tan^2\phi)^{3/2}}}$$



$$= \frac{2\pi\gamma}{\sqrt{\gamma^2-1}}\int_{-\phi_0}^{\phi_0}\cos\phi\, d\phi$$

$$= 2\pi\gamma\cdot \underline{2\sin\phi_0}.$$

$$= 2\pi\gamma\cdot 2\left(\frac{\sqrt{\gamma^2-1}}{\gamma}\right) \to \underline{4\pi q},$$