4/16/2008 Jackson (11.149)

$$\vec{E}' = \gamma(\vec{E} + \vec{\beta} \times \vec{B}) - \frac{\gamma^2}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{E})$$

$$\vec{B}' = \gamma(\vec{B} - \vec{\beta} \times \vec{E}) - \frac{\gamma^2}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{E})$$

$\vec{E} \parallel \vec{\beta}$. $\vec{\beta} \times \vec{E} = 0$ $\quad \vec{E}' = \gamma \vec{E} - \frac{\gamma^2 \beta^2}{\gamma+1} \vec{E}$

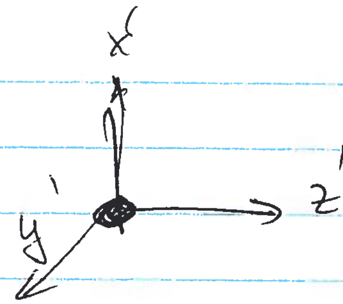
$$\gamma - \frac{\gamma^2 \beta^2}{\gamma+1} = \frac{\gamma(\gamma+1) - \gamma^2\beta^2}{\gamma+1} = \frac{\gamma^2(1-\beta^2) + \gamma}{\gamma+1} = 1.$$

$E'_{\parallel} = E_{\parallel}$ $\quad B'_{\parallel} = B_{\parallel}$

$$\vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \frac{\vec{v}}{c} \times \vec{B}_{\perp})$$

$$\vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \frac{\vec{v}}{c} \times \vec{E}_{\perp})$$

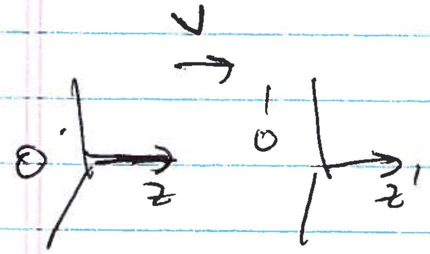
Point q



@ rest.

$$\vec{E}' = \frac{q\hat{r}'}{r'^2} = q\frac{\vec{x}'}{(r'^2)^{3/2}}$$

$$= q\left(\frac{x'\hat{x} + y'\hat{y} + z'\hat{z}}{(x'^2+y'^2+z'^2)^{3/2}}\right)$$



$x = x'$

$y = y'$

$z = \gamma(z' + vt')$

$t = \gamma\left(t' + \frac{z'v}{c^2}\right)$

inverse.

$z' = \gamma(z - vt)$

$t' = \gamma\left(t - \frac{vz}{c^2}\right)$

origins coincide @ $t = t' = 0$

$$E^z = E_\parallel = E'_\parallel = E'^{z'} = \frac{q\, z'}{(x'^2+y'^2+z'^2)^{3/2}} = q\frac{\gamma(z-vt)}{[x^2+y^2+\gamma^2(z-vt)^2]^{3/2}}$$

$$\vec{E}_\perp = \gamma\left(\vec{E}'_\perp + \frac{\vec{v}}{c}\times\cancel{\vec{B}'_\perp}\right) = \gamma\vec{E}'_\perp = \gamma\cdot\frac{q\,\vec{x}'_\perp}{(r')^3}.$$

$$E^x = \frac{\gamma q x}{(x^2+y^2+\gamma^2(z-vt)^2)^{3/2}} \qquad E^y \ \_\_\_\_\_ \ .$$

@ $t=0$

$$\boxed{\vec{E} = \frac{\gamma q(x\hat{x} + y\hat{y} + z\hat{z})}{(x^2+y^2+\gamma^2 z^2)^{3/2}}}$$

$z = r\cos\theta$

$$\vec{E} = \frac{\gamma q\,\vec{x}}{r^3\left(\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \gamma^2\cos^2\theta\right)^{3/2}}$$

$$\vec{E} = \frac{\gamma q \hat{r}}{r^2} \frac{1}{(\sin^2\theta + \gamma^2 \cos^2\theta)^{3/2}}$$

$$\vec{E} = q \frac{\hat{r}}{r^2} \frac{1}{\gamma^2} \frac{1}{\left((1-\frac{v^2}{c^2})\sin^2\theta + \cos^2\theta\right)^{3/2}}$$

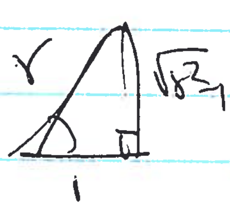
$$\boxed{\vec{E} = q \frac{\hat{r}}{r^2} \cdot \frac{1}{\gamma^2} \frac{1}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}} \qquad (11.154)$$

radial — inverse square — squeezed in direction of motion

$$\oint_{\text{sphere}} d^2a \cdot \vec{E} \cdot \hat{r} = \int r^2 d\Omega \, \frac{q}{r^2} \cdot \frac{1}{\gamma^2} \frac{1}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$

$$= q \int d\Omega \frac{\gamma}{(1+(\gamma^2-1)\cos^2\theta)^{3/2}} = 2\pi\gamma \int_{-1}^{1} d\mu \frac{1}{(1+(\gamma^2-1)\mu^2)^{3/2}}$$

$$(\gamma^2-1)\mu^2 = \tan^2\varphi \rightarrow 2\pi\gamma \int_{-\varphi_0}^{\varphi_0} \frac{\cancel{\sec^2\varphi}\, d\varphi}{\sqrt{\gamma^2-1}} \cdot \frac{1}{\cancel{(1+\tan^2\varphi)}^{3/2}}$$



$$= \frac{2\pi\gamma}{\sqrt{\gamma^2-1}} \int_{-\varphi_0}^{\varphi_0} \cos\varphi \, d\varphi$$

$$= 2\pi\gamma \cdot \underline{2\sin\varphi_0}$$

$$= 2\pi\gamma \cdot 2\left(\frac{\sqrt{\gamma^2-1}}{\gamma}\right) \rightarrow 4\pi q$$

Jumping ahead. §12.11 Covariant wave equation point charge source. 11.133 (12.123)

$$\boxed{\Box^2 A^\mu = \frac{4\pi}{c} J^\mu}$$

point particle.
$$\rho = e\,\delta^{(3)}(\vec{x} - \vec{r}(t))$$
$$\vec{J} = e\vec{v}\cdot\delta^{(3)}(\vec{x} - \vec{r}(t))$$

$$J^\mu(x) = ec\int d\tau\, U^\mu(\tau).\ \delta^{(4)}(x^\alpha - r^\alpha(\tau)).$$ (12.139) (14.2)

$$\delta(x^0 - r^0(\tau))\ \delta^{(3)}(\vec{x} - \vec{r}(\tau)).$$

use $\delta(x^0 - r^0(\tau))$ to do τ-integral

$$\delta(x^0 - r^0(\tau)) = \frac{\delta(\tau - (\tau_0))}{\left|\frac{dr}{d\tau}\right|_0} = \frac{\delta(\tau - \tau_0)}{\left|\frac{d}{d\tau}(ct)\right|}$$

$$= \frac{1}{\gamma c}\ \delta(\tau - \tau_0).$$

$$J^\mu(x) = (ec)\left(\frac{1}{\gamma c}\right)\left(\frac{\gamma c}{\gamma \vec{v}}\right)\delta^{(3)}(\vec{x} - \vec{r}(t(\tau_0)))$$

$$J^0 = ec.\ \delta^{(3)}(\vec{x} - \vec{r}(t))$$
$$\vec{J} = e\vec{v}\ \delta^{(3)}(\vec{x} - \vec{r}(t))$$

✓

(5)

$$\Box^2 A^\mu = \frac{4\pi}{c} J^\mu \qquad J^c$$

$$A^\mu(\vec{x},t) = A^\mu(x) = \int d^3x'\,dt' \; \frac{4\pi}{c}\, J^\mu(x') \; \frac{1}{4\pi}\frac{1}{|\vec{x}-\vec{x}'|}\; \delta\!\left(t'-t-\frac{1}{c}R\right)$$

e

$$\delta[(x-x')^2] = \delta\left[(x^0-x'^0)^2 - |\vec{x}-\vec{x}'|^2\right]$$

$$= \delta\left[(x^0-x'^0+R)(x^0-x'^0-R)\right]$$

$$= \frac{1}{2R}\delta(x^0-x'^0+R) + \frac{1}{2R}\delta(x^0-x'^0-R)$$

$x'^0 = x^0+R$ advanced. $\qquad$ $x'^0 = x^0 - R$ retarded.

$$G_{ret} = D_r = \frac{1}{2\pi}\delta[(x-x')^2]\,\Theta(x^0-x'^0) \qquad \underline{ct > ct'}$$

(12.133)

$$G_{adv.} = D_a = \frac{1}{2\pi}\delta[(x-x')^2]\,\Theta(x'^0-x^0) \qquad \underline{ct' > ct}$$

$$\boxed{A^\mu(x) = \int d^4x'\; D_r(x-x')\,\frac{4\pi}{c} J^\mu(x')}$$

(12.134)

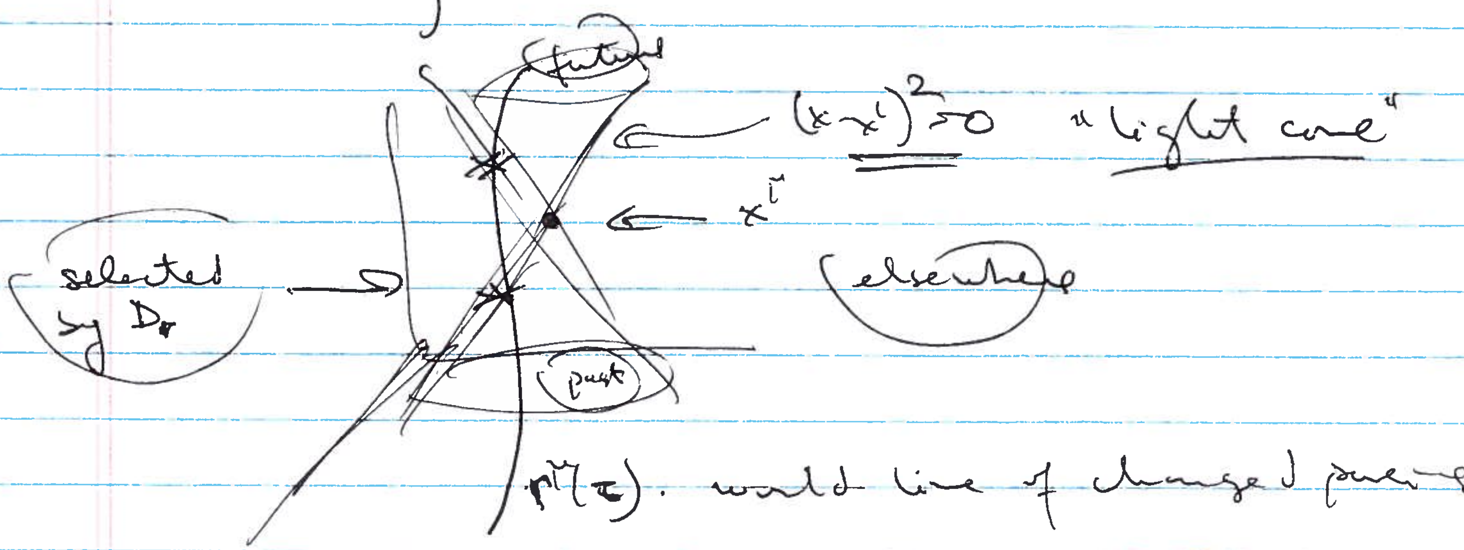
(14.1).

<u>Chapter 14.</u> Radiation by moving charges

$$A^\mu(x) = \int d^4x' \, \frac{1}{2\pi}\, \Theta(x^0 - x'^0)\, \delta\left[(x-x')^2\right]$$

$$\times \; e \cdot c \cdot \frac{4\pi}{c} \int d\tau \, V^\mu(\tau)\, \delta^{(4)}\left(x' - r(\tau)\right)$$

$$A^\mu = 2e \int d\tau \; V^\mu(\tau)\; \delta\left[(x - r(\tau))^2\right] \cdot \Theta\left(ct - r^0(\tau)\right)$$



$$\cancel{\delta\left[(x-r(\tau))^2\right] =}$$

$$\frac{d}{d\tau}\left[(x - r(\tau))^2\right] = \frac{d}{d\tau}\left[(x^0 - r^0(\tau))^2 - (\vec{x} - \vec{r}(\tau))^2\right]$$

$$= 2(x^0 - r^0)\left(-\frac{dr^0}{d\tau}\right) - 2\left(\vec{x} - \vec{r}(\tau)\right)\cdot\left(-\frac{d\vec{r}}{d\tau}\right)$$

$$= -2\,(x - r)\cdot\frac{dr}{d\tau} = -2\,V\cdot(x - r)$$

$$\boxed{A^\mu(x) = \frac{e\,V^\mu(\tau)}{V\cdot(x-r)}} \qquad (14.6).$$

$$x - r = \begin{pmatrix} c(t-t') \\ \vec{R} \end{pmatrix} \qquad V^\mu = \begin{pmatrix} \gamma c \\ \gamma \vec{v} \end{pmatrix} = \gamma c \begin{pmatrix} 1 \\ \vec{\beta} \end{pmatrix}$$

$$\boxed{\Phi(\vec{x}, t) = \left. \frac{e}{R(1 - \hat{n}\cdot\vec{\beta})} \right|_{t' = t - \frac{1}{c}R}} \qquad \boxed{\vec{A}(\vec{x}, t) = \left. \frac{e\vec{\beta}}{R(1-\hat{n}\cdot\vec{\beta})} \right|_{t'_{ret}}}$$

Famous. Liénard – Wiechert potential

1898 1901.

Engineer. Geophysicist.

Ecole des Mines de St Étienne 1892-1908 Göttingen

Chaire d'électricité industrielle

École des Mines de Paris 1908-1929.